M.A./M.Sc. Examination, 2019

Semester - IV

Mathematics

Optional Course: MMO-41(A05) (New Syllabus)

(Dynamics of Ecological System-II)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Notations and symbols have their usual meanings

Answer ***any four*** questions

1. a) Solve the nonlinear differential system

$$\frac{dx}{dt}=-x,\ \frac{dy}{dt}=-y+x^2,\ \frac{dz}{dt}=z+x^2$$

and determine the stable and unstable manifolds. 4

b) Sketch a phase portrait for the system

$$\frac{dx}{dt}=x+y-x\left(x^2+y^2\right),$$

$$\frac{dy}{dt}=-x+y-y\left(x^2+y^2\right),$$ 4

$$\frac{dz}{dt}=-z.$$

c) Prove that the origin of the system

$$\frac{dx}{dt}=-2y+yz,\ \frac{dy}{dt}=x(1-z),\ \frac{dz}{dt}=xy$$

is stable but not asymptotically stable by using the Lyapunov function $L(x, y, z)=ax^2+by^2+cz^2$. 2

2. Analyse the model and hence note down the effects of fear and additional food in the following delayed predator-prey model:

$$\frac{dx}{dt}=\frac{rx}{1+ky}-d_1x-a_1x^2-\frac{a_2xy}{b_1+\alpha\mu A+x},$$

$$\frac{dy}{dt}=\frac{ca_2\left[x(t-\tau)+\mu A\right]y}{b_1+\alpha\mu A+x(t-\tau)}-d_2y,$$ 5+5

$$x(t=0)\geq 0,\ y(t=0)\geq 0.$$

3. Consider the following modified Hastings and Powell model:

$$\frac{dX}{dT}=rX\left(1-\frac{X}{K}\right)-\frac{M_1(1-m_1)XY}{A_1+(1-m_1)X},$$

$$\frac{dY}{dT}=\frac{E_1M_1(1-m_1)XY}{A_1+(1-m_1)X}-D_1Y-\frac{M_2(1-m_2)YZ}{A_2+(1-m_2)Y}-HY^2,$$

$$\frac{dZ}{dT}=\frac{E_2M_2(1-m_2)YZ}{A_2+(1-m_2)Y}-D_2Z,$$

$$X(0)\geq 0,\ Y(0)\geq 0,\ Z(0)\geq 0,$$

$$0<E_1,\ E_2<1;\ m_1,\ m_2\in[0,1).$$

P.T.O.

Write down your observations around co-existence equilibrium point on the subsequent issues:
a) Boundedness,
b) Linear stability analysis,
c) Hopf-bifurcation,
d) Effect of refuge. 2+3+2+3

4. Carry out an analysis to obtain the necessary and sufficient conditions of Turing bifurcation for two species interaction with diffusion. 10

5. A reaction – diffusion model is given by

$$\frac{\partial M_1}{\partial t} = r\left(a - M_1 + M_1^2 M_2\right) + \frac{\partial^2 M_1}{\partial x^2},$$

$$\frac{\partial M_2}{\partial t} = r\left(b - M_1^2 M_2\right) + D\frac{\partial^2 M_2}{\partial x^2},$$

where M_1 and M_2 are morphogen concentrations. All the parameters r, a, b, c, D are positive constants.
a) Explain the model and find the non-zero homogeneous steady state.
b) Show the condition of stability (without diffusion) is $b - a - (a+b)^3 < 0$.
c) Linearize the system about the non-zero steady state and find the condition for diffusive instability. 2+3+5

6. Explicate the travelling wavefronts of pursuit and evasion in a predator-prey system:

$$\frac{\partial U}{\partial t} = AU\left(1 - \frac{U}{K}\right) - BUV + D_1\nabla^2 U,$$

$$\frac{\partial V}{\partial t} = CUV - DV + D_2\nabla^2 V,$$ 10

$$U(0, x) > 0, V(0, x) > 0, \nabla^2 \equiv \frac{\partial^2}{\partial x^2}.$$

______