B.A. (Honours) Examination, 2015

Semester – IV

Integrated Mathematics & Statistics

Paper : S.1.6 (Subsidiary)

Mathematics IV (Calculas-II)

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions

1. (a) Prove that the solution to any linear differential equation is equal to the sum of the homogenous solution and any particular solution to the complete differential equation. That is

   $y=y_h+y_p$

   (b) Solve the differential equation

   $\dot{y} = 0.1y - 1$

   And ensure that it satisfies the initial condition y(0)=5 at t=0. 5+5

2. (a) Prove that the solution to a linear, autonomous, first-order differential equation, $\dot{y} + ay = b$, converges to the steady-state equilibrium, $\bar{y} = \frac{b}{a}$, no matter what the initial value, y_0, if and only if the coefficient in the differential equation is positive: (a > 0).

   (b) Let y stand for energy demand, and suppose that it grows according to $\dot{y} = 5y - 10$.

   If energy demand has a value of 100 at time t=0, determine whether it ever converges to a steady state. 5+5

3. (a) Solve the following equations and ensure that the initial conditions are satisfied

   (i) $\dot{y} = 6y - 6$ and y(0) = 3

   (ii) $\dot{y} = 7$ and y(0) = 2

   (b) Let p(t) represent the consumer price index. If the rate of inflation of the price index is constant at 5% (i.e. the growth rate of p(t) is 5%), and the price index has a base value of 100 at time t = 0, solve for the expression showing price index as a function of time. (3+3)+4

4. Find the steady state equilibrium point for the Neo-classical Model of Economic Growth (Solow Model). 10

P.T.O.

5. (a) Find the steady-state points and determine their stability properties for the following:

$$\dot{y} = y^{1/2} - y.$$

(b) Increase in carbon dioxide in the earth's atmosphere have been cited as a probable cause of "global warming". Let y represent the stock of cabon dioxide and let x>0 (a constant) represent the flow of carbon dioxide emissions that come from industrial activity. Assume that the dynamics of y are given by

$$\dot{y} = x - y^a$$

Where the term y^a represents the earth's capacity to remove carbon dioxide from atmosphere and allow its absorption elsewhere (i.e., in trees, oceans). Conduct a qualitative analysis of this model, first for the case $a > 0$ and then for the case $a < 0$. Comment on your results. 4+6

6. Solve the following equations using initial conditions y_0=10 and $\dot{y}_0 = 8$:

(a) $\ddot{y} + \dot{y} + \frac{1}{4}y = 2$

(b) $2\ddot{y} - \frac{1}{3}\dot{y} - \frac{1}{3}y = 10$ 5+5

7. (a) Prove that, if y_1 and y_2 are two distinct solutions to the differential equation $\ddot{y} + a_1 y + a_2 y = 0$ and c_1 and c_2 are any two constants, then the function $y=c_1y_1+c_2y_2$ is solution to the above differential equation. Conversely, if y is any solution to the above mentioned differential equation, then there are unique constants, c_1 and c_2, such that $y=c_1y_1+c_2y_2$.

(b) Classify the following equation according to: (i) order, (ii) linear or nonlinear, (iii) autonomous or non-autonomous, and (iv) difference or differential equation

$$\dot{y} = 2y^3 + t.$$ 7+3

8. Write short note on the following:

(a) Separable Equations.

(b) Bernoulli's Equation. 5+5