B.A. (Honours) Examination, 2015
Semester – II
Integrated Mathematics & Statistics
Paper : S.1.3 (Subsidiary)
Mathematics (Algebra-II)

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions 4×3=12

1. (i) Express $(1+i)^2/(3-i)$ in the form $(A+iB)$.

   (ii) Find 'z' in the form of $(A+iB)$, so that $z.(3+4i)=2+3i$

   (iii) Show that $\sin x$ is a is a periodic function of x with the periodicity $2n\pi$ (n=positive integer).

   (iv) Find the quotient and the remainder when $(3x^4+5x^3-2x^2+4x+6)$ is divided by $(x-1)$.

   (v) Show that the equation $x^4+15x^2+7x-10=0$ has at least two imaginary roots.

   (vi) The roots of the equation $x^3-px^2+qx-r=0$ are in G.P. Prove that $q^3=p^3.r$.

   (vii) Suppose $A=\{x: x\in N, x<8, x \text{ is odd}\}$

   and $B=\{x: x\in N, x \text{ is divisible by } 5\}$

   Find (A–B) and the power set of (A–B).

   (viii) Find whether the relation R as defined below in the set $A=\{1,2,3\}$ is reflexive, symmetric and transitive.

   $$R=\{(2,1), (1,2), (3,3)\}$$

2. Answer ***any four*** questions 4×4=16

   (i) Find the cube root of '–1'.

   (ii) Show that the equation $\tan\left(i\log\dfrac{x-iy}{x+iy}\right)=2$ represents the rectangular hyperbola $x^2-y^2=xy$.

   (iii) From the definition of e^z, express $\sin x$ and $\cos x$ in terms of e^{ix} and e^{-ix}.

   (iv) Find the value of $\left(x^3-7x^2-2x+88\right)$ when $x=5+i\sqrt{3}$.

P.T.O.

(v) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then find the equation whose roots are $\alpha^2\beta^2, \alpha^2\gamma^2$ and $\beta^2\gamma^2$

(vi) Remove the second term of the equation

$$x^4 + 8x^3 + x - 5 = 0$$

(vii) A binary operation '*' is defined such that (a*b) gives the remainder of (a+b) divided by 3. If a set is defined as S={0,1,2}, then show that 0 is the identity element with respect to the operation '*'.

(viii) Show that the following four matrices form a group under matrix multiplication.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

3. Answer ***any two*** questions $2\times6=12$

(i) State De Moivre's theorem. Prove the theorem for 'n' being a positive integer.

(ii) Solve $x^3 - 6x - 9 = 0$ by Cardan's method.

(iii) Consider the set S={1,2,3}. Write down all the permutations that are possible with the elements of S. Show that these permutations form a group with the operation permutation multiplication.

(iv) Define a group and an abelian group. Show that the set of cube roots of unity is an abelian group with respect to multiplication.