Use separate answer Script for each Unit

M.A./M.Sc. Examination 2019

Semester - IV

Mathematics

Elective Course: MME-41(Pure Stream)

(Galois Theory-II and Algebraic Topology)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Galois Theory-II)

Answer ***any four*** questions.

1. Give an example with justification of an extension $F/\mathbb{Q}$ which is normal. Let F/K be a finite extension. If it has only finitely many intermediate fields then show that F/K is simple. 2+3

2. Let F/K be a finite extension. Prove that $\left|G\left(F/K\right)\right| \le [F:K]$. Hence or otherwise find $G\left(\mathbb{C}/\mathbb{R}\right)$. 3+2

3. Show that $G\left(\mathbb{Q}\left(\sqrt{2}, \sqrt{3}\right)/\mathbb{Q}\right) \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$. Consider a subgroup H of $G\left(\mathbb{Q}\left(\sqrt{2}, \sqrt{3}\right)/\mathbb{Q}\right)$ of order 2 and find F_H. 3+2

4. Show that every Galois extension is a splitting field of a separable polynomial. State the fundamental theorem of Galois theory. 3+2

5. Let F/K be a finite extension and H be a subgroup of $G\left(F/K\right)$. Prove that $|H| \ge [F:F_H]$. Hence or otherwise show that $H = G\left(F/F_H\right)$. 4+1

6. Define constructible real numbers. Show that the set k of all constructible real numbers is a field. Also prove that it is impossible to construct a cube with a volume 2 unit^3, using only ruler and compass. 1+3+1

Unit-II (Algebraic Topology)

Answer ***any two*** questions.

1. a) Define a contractible space. Show that $(n+1)$ disc E^{n+1} is contractible. 1+1

   b) Show that a space is contractible if and only if it has the same homotopy type as an one point space. 2

   c) Show that a map $f: s^n \to Y$ can be extended continuously over E^{n+1} if and only if f is null homotopic. 3

   d) Let (X, A) has the homotopy extension property with respect to Y and $f_0, f_1 : A \to Y$ are homotopic. If f_0 has an extension over X, then show that f_1 has also an extension over X. 3

P.T.O.

2. a) If (X, A) has the homotopy extension property with respect to A, then show that A is weak retract of X iff A is retract of X. 2

   b) Show that a space X is deformable into a subspace A of X if and only if the inclusion map $i : A \to X$ has a right homotopy inverse. 3

   c) If any two maps f_0 and f_1 of any space X into S^n are such that $f_0(x) \neq -f_1(x)$ for any $x \in X$, then show that $f_0 \simeq f_1$. 2

   d) Show that the relation between maps from (X, A) to (Y, B) of being homotopic relative to A is an equivalence relation. 3

3. a) Define a fundamental group. If X is path connected and x_0, x_1 are two points of X, then show that $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$. 1+3

   b) Show that the map $p : \mathbb{R} \to S^1$ given by the equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map. 3

   c) Show that the fundamental group of S^1 is isomorphic to the additive group of integers. 3

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