M.A./M.Sc. Examination 2019

Semester - IV

Mathematics

Course: MMC-42(New & Old Syllabus)

(Operations Research)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions.

1. a) When an LPP has unbounded solution? Give the answer in connection with the revised simplex method (RSM). 2

   b) Find an optimum solution to the following LPP by applying RSM:

   $$\text{Maximize } f(x_1, x_2) = x_1 + x_2$$
   $$\text{subject to } 2x_1 + 5x_2 \geq 6,$$
   $$x_1 + x_2 \geq 2,$$
   $$x_1, x_2 \geq 0.$$ 8

2. a) Briefly discuss the utility of sensitivity analysis. 2

   b) Consider the following LPP:

   $$\text{Maximize } f(x_1, x_2) = 4x_1 + 5x_2$$
   $$\text{subject to } 3x_1 + 4x_2 \leq 14,$$
   $$2x_1 + x_2 \leq 6,$$
   $$4x_1 + 2x_2 \leq 8,$$
   $$x_1, x_2 \geq 0.$$

   Show that the optimality of the solution is not violated if the right hand side of the first constraint varies between 6 and 16. Show further that the range of c_2 is $\left(2, \frac{16}{3}\right)$ in order that the optimal solution obtained remains optimal. 4+4

3. a) State the relative advantages and disadvantages of the Gomory's cutting plane method over the Branch and Bound method to solve an integer linear programming problem (ILPP). 2

   b) Solve the following ILPP:

   $$\text{Maximize } f(x_1, x_2) = 6x_1 + 2x_2$$
   $$\text{subject to } 3x_1 + 2x_2 = 5,$$
   $$x_1, x_2 \geq 0 \text{ and are integers.}$$ 8

4. a) Use the Kuhn-Tucker necessary and sufficient conditions to solve the following nonlinear programming problem (NLPP):

   $$\text{Minimize } f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 5)^2$$
   $$\text{subject to } -x_1^2 + x_2 \leq 4,$$
   $$x_1 - (x_2 - 2)^2 \leq 3,$$
   $$x_1, x_2 \geq 0.$$ 5

P.T.O.

b) Solve the following NLPP graphically:

$$\text{Maximize } f(x_1, x_2) = x_1 + 2x_2$$
$$\text{subject to } x_1^2 + x_2^2 \le 1,$$
$$2x_1 + x_2 \le 2,$$
$$x_1, x_2 \ge 0.$$

5

5. a) In a deterministic inventory model where supply is instantaneous and shortages are permitted in the sense of back orders, prove that the optimal value of total inventory cost is $\sqrt{\frac{2DC_S C_1 C_2}{C_1 + C_2}}$. 6

b) The demand for an item is 18,000 units per year. The holding cost per unit time is Rs. 1.20, the cost of shortage is Rs. 5.00 and the production cost is Rs. 400.00. Assuming that replenishment rate is instantaneous, determine the optimal order quantity. 4

6. a) For the queuing model system $(M/M/1:N/FCFS)$, find the probability of n customers in the system in the form $p_n = \frac{(1-\rho)\rho^n}{1-\rho^{N+1}}$, $\rho \ne 1$, $n = 0, 1, 2, ..., N$. 6

b) A car park contains 5 cars. The arrival of car is Poisson at a mean rate of 10 per hour. The length of time each car spends in the car park has negative exponential distribution with mean of 5 hours. How many cars are in the car park and what is the probability of a newly arriving customer finding the car park full and having to park elsewhere? 2+2

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