M.A./M.Sc. Examination 2019

Semester - IV

Mathematics

Course: MMC-41 (Old Syllabus)

(Differential Geometry)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Notations and symbols have their usual meanings.

*Answer **any four** questions.*

1. a) Show that $\left\{ \begin{matrix} i \\ ij \end{matrix} \right\} = \frac{\partial}{\partial x^j}\left(\log\sqrt{g}\right)$, where $g = \left|g_{ij}\right|$. 5

   b) Show that $g_{ik,\,j} = 0$, $g^{ik}_{,\,j} = 0$ and $\delta^i_{k,\,j} = 0$. 5

2. a) Define curvilinear coordinate system. Show that $\left(x^1, x^2, x^3\right)$ defined by the following transformations:

   $$y^1 = x^1 \sin x^2 \cos x^3$$
   $$y^2 = x^1 \sin x^2 \sin x^3$$
   $$y^3 = x^1 \cos x^2$$

   are curvilinear coordinates and also find the coordinate surface. 1+4

   b) Prove that $\frac{d}{dt}\left(g_{ij}A^iB^j\right) = g_{ij}\frac{\delta A^i}{\delta t}B^j + g_{ij}A^i\frac{\delta B^j}{\delta t}$. 5

3. a) Find the elementary arc length of a rectilinear in spherical polar coordinate system. 5

   b) Prove that a curve with vanishing torsion and with non-zero constant curvature is a circle. 5

4. a) Establish Serret-Frenet formulae. 8

   b) If K is the curvature of a curve, prove that

   $$K^2 = g_{ij}\frac{\delta t^i}{\delta s}\frac{\delta t^j}{\delta s}.$$ 2

5. a) Show that the ratio of the curvature to the torsion of a space curve is a non-zero constant iff the curve is a helix. 5

   b) A curve C is defined in cylindrical coordinates x^i as follows:

   $$x^1 = a,\ x^2 = \theta,\ x^3 = b\theta\left(b \neq 0\right)$$

   where a, b are constants in which 'a' is positive and θ is a function of s. Prove that $k = \frac{a}{a^2+b^2}$, $\tau = \frac{b}{a^2+b^2}$. 5

6. a) Prove that $\sigma^2 = a_{\alpha\beta}\frac{\delta t^\alpha}{\delta s}\frac{\delta t^\beta}{\delta s}$ and $\sigma = \varepsilon_{\alpha\beta}t^\alpha\frac{\delta t^\beta}{\delta s}$. 5

   b) Show that the Gaussian curvature under any suitable transformation is invariant. 5

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