

M.A./M.Sc. Examination, 2019
Semester - II
Mathematics
Core Course: MMC-26 (Old Syllabus)
(Numerical Analysis)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.

Answer *any four* questions

1. a) Derive the Hermite interpolation formula for the function $f(x)$ whose tabular values along with the values of its first order derivative at $(n+1)$ distinct points are given. Comment on its advantages over other usual interpolation formulae. 6
b) Show that the divided difference of order n can be expressed as a quotient of two determinants each of order $(n+1)$. 4
2. a) Obtain the best uniform polynomial approximation $P_n(x)$ of degree n in the minimax sense to the function $f(x)$ which is continuous on $[a, b]$. 6
b) Approximate e^x by taking first two terms of the Taylor's series expansion about $x = \alpha$ on $[0, 1]$. How should α be chosen so as to minimize the error in maximum norm? 4
3. a) Explain briefly the Romberg's quadrature rule for numerical integration of $f(x)$ over the interval $[a, b]$. Point out the stopping criterion based on either a row or a column convergence test. Comment also on the accuracy of the method. 6
b) Determine the coefficients in the formula
$$\int_0^{2h} x^{-1/2} f(x) dx = (2h)^{1/2} [A_0 f(0) + A_1 f(h) + A_2 (2h)] + R$$
And calculate the remainder R , when $f'''(x)$ is constant. 4
4. a) Explain briefly the power method for the largest eigen pair of a real numerical matrix including its stopping criterion. State how the least eigen pair can be obtained for both the cases of singular and non-singular matrices and also how other remaining eigen pairs are obtained. 6
b) Using a similarity transformation, find A^p where
$$A = \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$$
and p is a positive integer. 4
5. a) Carry out an error analysis to discuss the convergence of a single-step method used to solve an well-posed initial value problem $\frac{dy}{dx} = f(x, y)$, $y(a) = y_0$ in a finite interval $[a, b]$. 6
b) Prepare a scheme corresponding to the single-step method for the differential equation $\frac{dy}{dx} = f(x, y)$ which produces exact result for $y(x) = a + be^{-x}$; a, b being constants. 4

P.T.O.

6. a) Briefly describe an explicit finite difference scheme for solving the problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c > 0, \quad t > 0, \quad 0 < x < l$$

with $u(0, t) = 0 = u(l, t), \quad t > 0$

$$u(x, 0) = F(x) \quad \text{and} \quad \frac{\partial u}{\partial t}(x, 0) = G(x), \quad 0 \leq x \leq l$$

in which $F(x)$ and $G(x)$ are known functions of x .

6

- b) Discuss the shooting method for solving boundary value problem

$$-u'' + p(x)u' + q(x)u = r(x), \quad a < x < b$$

subject to the given conditions

$$u'(a) = \gamma_1, \quad u'(b) = \gamma_2$$

where $p(x)$, $q(x)$ and $r(x)$ are assumed to be continuous on $[a, b]$ and γ_1, γ_2 are non-zero real constants.

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