M.A./M.Sc. Examination, 2019

Semester - II

Mathematics

Core Course: MMC-26 (New Syllabus)

(Numerical Analysis)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions

1. a) What is the basic difference between ordinary interpolation and Hermite interpolation? Which one is more accurate and why? State the properties of the general spline function. When does it call a natural spline? Indicate the merits and demerits of both quadratic and cubic spline and state the order of convergence of a cubic spline. 6

   b) Write down the general divided difference interpolation formula for $(n+1)$ distinct points $(x_i, y_i), i = 0(1)n$. Derive the corresponding formula due to Lagrange right from this formula. 4

2. a) Using Gram-Schmidt orthogonalisation process, find the least squares polynomial approximation of degree *n* for the function $f(x)$ having $(n+1)$ tabular values corresponding to distinct argument values with a non-negative weight function $w(x)$. 6

   b) Find the constants $a_k\ (k = 0, 1, 2)$ so that $\sum_{n=0}^{2} a_n T_n(x)$ is a good approximation to $(1+x)^{-1}$ for $0 \le x \le 1,$ in which $T_n(x) = \cos n\theta$ where $\cos\theta = 2x - 1$. 4

3. a) Explain briefly the Romberg's quadrature rule for $\int_a^b f(x)dx$. Point out the stopping criterion based on either a row or a column convergence test. Comment also on the accuracy of the method. 6

   b) State and prove the necessary and sufficient condition for the degree of precision of Gauss quadrature formula to be $(2n+1)$. 4

4. a) Briefly describe Givens' method for computing all eigen pairs of a real symmetric numerical matrix of order *n*. 6

   b) The matrix $A = \begin{pmatrix} 1+s & -s \\ s & 1-s \end{pmatrix}$ is given. Calculate *p* and *q* such that $A^n = pA + qI$ and hence show that $e^A = eA$. 4

5. a) Carry out an error analysis to discuss the convergence of a single-step method used to solve an well-posed initial value problem $\frac{dy}{dx} = f(x, y),\ y(a) = y_0$ in a finite interval [*a, b*]. 6

   b) Make a comparative statement regarding the criterion of accuracy of both the Adams' and Milne's predictor-corrector method right from their respective formulae. Comment further on the stability of the methods. 4

P.T.O.

6. a) Briefly describe an explicit finite difference scheme for solving the problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},\ c > 0,\ t > 0,\ 0 < x < l$$

with $u(0, t) = 0 = u(l, t),\ t > 0$

$$u(x, 0) = F(x) \text{ and } \frac{\partial u}{\partial t}(x, 0) = G(x),\ 0 \le x \le l$$

in which $F(x)$ and $G(x)$ are known functions of x. 6

b) Discuss the shooting method for solving boundary value problem

$$-u'' + p(x)u' + q(x)u = r(x),\ a < x < b$$

subject to the given conditions

$$a_0 u(a) - a_1 u'(a) = \gamma_1,\ b_0 u(b) + b_1 u'(b) = \gamma_2$$

where $a_0, b_0, a_1, b_1, \gamma_1$ and γ_2 are non-zero constants; $p(x), q(x)$ and $r(x)$ are assumed to be continuous on [*a, b*]. 4

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