M.A./M.Sc. Examination, 2019

Semester - II

Mathematics

Core Course: MMC-25 (Old Syllabus)

(Dynamical Systems)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

(Notations and symbols have their usual meaning unless otherwise stated)

Answer ***any four*** questions

1. a) Find e^{At} for the system $\dot{X} = AX$, where $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$. 3

   b) If $P^{-1}AP = D$, where D is a diagonal matrix then show that $e^{At} = Pe^{Dt}P^{-1}$ where A is a matrix. 3

   c) Linearize the system $\dot{x} = e^{-x-3y} - 1$ and $\dot{y} = -x\left(1 - y^2\right)$ about the fixed point (0, 0). Also classify the fixed point. 4

2. a) Draw the phase portrait of the system $\ddot{x} + \dot{x} = ax^2$ for $a = 0, -1$ and $+1$. 3

   b) Prove that a gradient system cannot have a closed orbit. 3

   c) Show that the system

   $$\dot{x} = -y + x\left(\sqrt{x^2 + y^2} - 1\right)\left(2 - \sqrt{x^2 + y^2}\right)$$

   $$\dot{y} = x + y\left(\sqrt{x^2 + y^2} - 1\right)\left(2 - \sqrt{x^2 + y^2}\right)$$

   has exactly two limit cycle. 4

3. With the help of phase diagram and bifurcation diagram discuss the bifurcation of the following systems:

   a) $\dot{x} = \mu - x^3$

   b) $\dot{x} = \mu x - x^2$

   c) $\dot{x} = \mu + x^2$

   $\mu \in R$ 10

4. a) Show that the system

   $$\dot{x} = -y\left(x^2 + y^2\right)^{1/2} \qquad \dot{y} = x\left(x^2 + y^2\right)^{1/2}$$

   is orbitally stable but not Lyapunov stable. 4

   b) Using Hartmann-Grabmann theorem discuss the local stability of the equilibrium point for the nonlinear system $\dot{x} = x - y^2$, $\dot{y} = y$. Also find the Homeomorphic map. 3+3

5. a) Define topological conjugacy and topological semi-conjugacy. Show that the tent map is topologically conjugate to the map $Q(x) = 1 - 2x^2$, $x \in [-1, 1]$. 2+3

   b) Prove that the logistic map $f(x) = rx(1 - x)$ has a 2-cycle for all $r > 3$. Also show that the 2-cycle of the logistic map is linearly stable when $3 < r < \left(1 + \sqrt{6}\right)$. 5

P.T.O.

6. a) Define Lyapunov exponent of a map. Hence find its expression. 2.5
   b) Show that a doubling map $g : S \to S$ defined by $g(\theta) = 2\theta$ is chaotic on the unit cycle S. 2.5
   c) Find the Lyapunov exponent for the Tent map. 2.5
   d) Explain SDIC and Topological transitivity. 2.5

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