Use separate answer script for each Unit

M.A./M.Sc. Examination, 2019

Semester - II

Mathematics

Core Course: MMC-25 (New Syllabus)

(Solid Mechanics and Dynamical Systems)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

(Notations and symbols have their usual meaning unless otherwise stated)

Unit-I [Solid Mechanics (Marks: 20)]

Answer ***any two*** questions

1. a) Prove that the extremum values of normal strain at a point of a continuum are principal strains. 5

   b) For the deformation $u_i = A_{ij}X_j$, where A_{ij} are constants $(i, j = 1, 2, 3)$, determine an expression for change of volume per unit original volume. If A_{ij} are very small, show that it reduces to cubical dilatation. 5

2. a) Derive stress equation of equilibrium. 5

   b) Stress tensor at a point is given by

$$(\tau_{ij}) = \begin{pmatrix} 5 & 0 & 0 \\ 0 & -6 & -12 \\ 0 & -12 & 1 \end{pmatrix}.$$

   Determine maximum shear stress. 5

3. a) State the principle of conservation of mass in spatial method and hence obtain the equation of continuity. 2+3

   b) Obtain the geometrical interpretation of the infinitesimal shear strain components. 5

Unit-II [Dynamical Systems (Marks: 20)]

Answer ***any two*** questions

1. a) Show that the linear harmonic oscillator is orbitally stable in the sense of Poincare but not asymptotic stable. 2

   b) State stable manifold theorem. Find the local stable and unstable manifolds of the system $\dot{x} = x - y^2,\ \dot{y} = -y.$ 1+2+2

   c) Define topological conjugecy. Show that the tent map $T(x)[0, I] \to [0, 1]$ is topologically conjugate to the logistic map $x_{n+1} = 4x_n(1 - x_n)\ x_n \in [0, 1]$ through the conjugecy map $h(x) = (1 - \cos \pi x)/2.$ 1+2

2. a) Explain the stability behavior of the fixed points of the quadratic map $\mathbb{Q}_c(x) = x^2 + c\ \forall\ x \in \mathbb{R}.$ 2

   b) Define Schwarzian derivative $Sf(x)$. If $f(x)$ be a polynomial such that all roots of $f'(x)$ are real and distinct, then show that $Sf(x) < 0.$ 1+2

   c) State Lienard theorem. Hence explain the stability criterion of the Van-der-Pol equation. 1+2

P.T.O.

d) Use Bendixson's negative criterion, show that the system

$$\dot{x} = -y + x\left(x^2 + y^2 - 1\right)$$
$$\dot{y} = x + y\left(x^2 + y^2 - 1\right)$$

has no closed orbit inside the circle with centre (0, 0) and radius $\frac{1}{\sqrt{2}}$. 2

3. a) Show that the system

$$\dot{x} = -y + x\left(x^2 + y^2\right)\sin\left(\log\sqrt{x^2 + y^2}\right)$$
$$\dot{y} = x + y\left(x^2 + y^2\right)\sin\left(\log\sqrt{x^2 + y^2}\right)$$

has infinite number of periodic orbits. 3

b) Prove that the logistic map $f(x) = rx(1-x)$ has a two cycle $\forall\ r > 3$. 3

c) Find the fixed points of $S^2(x)$ when $S:[0,1) \to [0,1)$ is defined as $S(x) = 2x(\text{mod } 1)$. Hence show that $\left\{\frac{1}{3}, \frac{2}{3}\right\}$ forms a period-2 cycle for $S^2(x)$. 2+1

d) Show that ± 1 are super stable fixed points of the map $f(x) = \dfrac{3x - x^3}{2}$, $x \in R$. 1