

M.A./M.Sc. Examination, 2019
Semester - II
Mathematics
Core Course: MMC-24 (New Syllabus)
(Classical Mechanics)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.
(Notations have their usual meaning unless otherwise stated)
Answer *any eight* questions

1. Derive the approximate equation of motion of a particle relative to an observer placed at a point with colatitudes λ on the earth surface (taking into account the rotation of the earth about its axis only). A projectile is fired with a high initial velocity v_0 in a nearly horizontal direction (eastward, may be assumed to be y -axis). Show that the projectile will be deflected to left or right to determined by you. What happen if the one neglects the rotation of the earth about its axis. 2+2+1
2. Derive the expression for angular momentum and kinetic energy of a rigid body due to its rotational motion about a point fixed in the body. Hence derive the Euler's equation for the space motion of a symmetric rigid body. 2+1+2
3. Explain invariable plane, space and body cones involved with space motion of a symmetric rigid body. 2+1+2
4. Define Euler's angle variables. Derive expressions for the components of angular velocity about axes fixed in the body in terms of Euler's angle variables and their derivatives. 2+3
5. Derive the expression for kinetic energy of system of N -particles following some rhenomic holonomic constraints in terms of generalized coordinates and their time derivatives. Derive the expression for kinetic energy for the motion of a particle in the space satisfying the constraints $x^2 + y^2 + z^2 = a^2$. 2.5+2.5
6. Define Poisson bracket and state their properties. Prove that the Poisson bracket of two constant of the motion is also a constant of the motion. Verify whether the transformations $(q, p) \rightarrow (Q, P) = (\sqrt{2p \sin q}, \sqrt{2p \cos q})$ among the phase space variables are canonical. 2+1+2
7. What do you mean by normal coordinates and normal frequency of small oscillations of mechanical system changing state around one of its stable equilibrium? Find normal coordinates and normal frequency of the system with Hamiltonian $H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + \frac{k}{2}(y-x)^2 + \frac{k}{2}(z-y)^2 + \frac{k}{2}(z-x)^2$, if any. 2+3
8. Define cyclic coordinate. Show that for some mechanical system there exists a function which may play the dual role viz. Lagrangian and Hamiltonian of the system. Give some examples in support of your answer. 1+3+1
9. State Hamilton's principle. Derive Lagrange's and Hamilton's equation of motion from it. Derive the Lagrangian for the motion of a heavy particle hanging from a point in the roof of your room oscillating around its stable equilibrium. 1+2+2

P.T.O.

10. Define Hamilton's principle function. State the difference between Hamilton's principal function and the action integral involved in the Hamilton's principle. Find Hamilton's principal function for the harmonic oscillator problem. 1+1+3
 11. State Noether's theorem. Write down transformations: translation in space, time and rotation of space coordinates in their infinitesimal form. Find the conserved quantity corresponding to these transformations. 1+2+2
 12. Show that it is possible to choose a special kind of generating function for a canonical transformation which may become Hamilton's principal function for a mechanical system. Further show that such may help one to get the solution of Hamilton's equation of a system involving n -generalized coordinates by solving a single first order partial differential equation instead of solving $2n$ first order ordinary differential equations. 2+3
 13. What are action-angle variables? For which mechanical system such coordinates exists? Derive the evolution of those variables, whenever the system admits action-angle variables. 0.5+0.5+4
 14. State principle of quantum mechanics and derive the evolution equation of the probability amplitude involved there. Interpret the equation. State Feynman's principle of democratic equality. Derive the expression for the kernel involved in the evolution equation for probability amplitude mentioned above. 3+0.5+0.5+1
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