M.A./M.Sc. Examination, 2019

Semester - II

Mathematics

Core Course: MMC-22 (Old Syllabus)

(Topology)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions

1. a) State and prove Well ordering theorem. 5

   b) Prove that Zorn's Lemma (Restricted form) $\Rightarrow$ Hausdorff maximality principle $\Rightarrow$ Zorn's Lemma (General form). 5

2. a) Let *X* be an infinite set and $\tau = \{\phi\} \cup \{G : G \subset X \text{ and } G^C \text{ is finite}\}$. Show that τ is a topology on *X*. 4

   b) Show that the upper limit topology u for $\mathbb{R}$ is finer than the usual topology τ for $\mathbb{R}$. 2

   c) Prove that the intersection of any family of topologies for *X* is a topology for *X*. 4

3. a) Let (*X*, □) be an indiscrete topological space and *A* be any subset of *X*. Find the derived set of A. 4

   b) Find a necessary and sufficient condition for a family of subsets of *X* to be a base for some topology on *X*. 6

4. a) When is a topological space said to be first countable. Prove that a second countable topological space is first countable. Show by an example that converse need not be true in general. 6

   b) Let (X, τ) be a first countable topological space. Then show that there exists a nested local base at every point of x of X. 4

5. a) Let (X, τ) and (Y, τ^*) be two topological spaces. Let $f : X \to Y$ be a single valued mapping. Then show that f is an open mapping iff $f[Int\, A] \subset Int\, f[A]$ for every $A \subset X$ and a closed mapping iff $Cl[f[A]] \subset f[Cl(A)]$ for every $A \subset X$. 6

   b) Show that Lindelöffness is not a hereditary property for a topological space. 4

6. a) A topological space *X* is disconnected if and only if there exists a continuous mapping of *X* onto the two pointic discrete space $Y = \{0, 1\}$ – prove it. 5

   b) Prove that a topological space *X* is normal if and only if for any closed set $F \subset X$ and an open set $G \subset X$ containing *F*, there exists an open set *V* such that

   $$F \subset V \subset \overline{V} \subset G.$$ 5
