M.Sc. Semester-II Examination 2017
Statistics (New)
Course : MSC-21
(Inference-II)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. Discuss the role of weight function in multiparameter testing problem. Using this or otherwise, find UMP test for $H: \mu_1 \le \mu_2$ against K: $\mu_1 > \mu_2$ when observations are drawn from independent $N(\mu_i, 1); i = 1, 2$. 3+7=10

2. (a) Let $(x_1, x_2, ..., x_n)$ be iid with a common density $p(x \mid \theta) = e^{-(x-\theta)}; x > \theta, -\infty < \theta < \infty$. Show that $\{p(x \mid \theta); -\infty < \theta < \infty$ has MLR in $X_{(1)}$.

   (b) Show that for every $0 < \alpha < 1$ and for some statistic T=T(X), we can always find a right-tailed size-α test based on T for testing $H: \theta = \theta_0$. 5+5=10

3. Let $(x_1, x_2, ..., x_N)$ be iid with a common exponential density $p(x \mid \theta) = \theta e^{-\theta x}; x > 0, \theta > 0$. Find a UMP size-α test for: (a): $\theta = \theta_0$ against $K: \theta > \theta_0$ (b) $H: \theta \le \theta_0$ against $K: \theta > \theta_0$. 7+3=10

4. Define MLR family. Suppose X has a binomial (n, p) distribution, where p is unknown. Show that X belongs to MLR family. Hence find an UMP size α test for $H_0: p \le p_0$ against $H_1: p > p_0$. Discuss the nature of the power function of the test. 2+3+3+2=10

5. Describe the median test for testing the identity of two univariate continuous distributions against the alternative that the location of one distribution is shifted to the right of the other. Find the null distribution of the test statistic and establish the symmetry of this distribution. Also find its mean and variance. 10

6. Consider the linear rank statistic $T_N = \sum_1^N a_i z_i$ where z_i's have discrete uniform destination over $\{1, 2, ...N\}$. If $\{\alpha_i\}$ satisfies the condition $a_i + a_{n-i+1}$ = constant, prove that the distribution of T_N is symmetric. Derive mean and variance of T_N. 10

7. Write short notes on **any two** of the following : 5+5=10

   (a) Wilcoxon Test

   (b) One parameter exponential family and MLR property

   (c) Signed rank test
