M.A. Examination, 2014

Semester – III

Economics

Paper – X

(Quantitative Methods)

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. Let aggregate output in an economy be given by

$$y_t = k_t^{\alpha}, \quad 0 < \alpha < 1, \quad t = 0,1,2,3$$

Where y_t is aggregate output and k_t is aggregate capital stock. Capital is accumulated by saving a constant share s = 0.2 of output each period. Assuming further that capital depreciates at the rate

$\delta = 0.02$ we have

$$k_{t+1} = k_t - \delta k_t + \delta y_t$$

If α=0.8 and the initial value of capital is k_0=95000

Find (1) the Steady State Value of the capital stock and (2) determine whether k_t converges to it in a monotonic or in an oscillalony fashion. 3+7

2. Suppose the growth of fishes in a fishery in the absence of any fishing is given by the function $g(y) = y - y^2$

Explicitly solve for the fish population as a function of t when the rate of fishing is $h = \alpha y$ 10

3. An individual's **instantaneous utility** is function of her consumption

$$U(c(t)) = \ln c(t)$$

Where $c(t)$ denotes the money withdrawn from her bank account, the initial balance is $x(0) = x_0$ and the differential x equation for the bank account is $\dot{x} = \frac{dx}{dt} = rx - c$

Find the consumption path that maximized her **lifetime utility**

$$u = \int_0^T e^{-pt} U(c(t))dt$$

Over $(0,T)$ where $\rho \geq 0$ her is personal rate of time preference, subject to the condition that she must leave an amount of money ***b*** in her bank account at $T, x(T) = b > 0$ 10

4. Solve the free endpoint, infinite time investment model

$$\max \int_0^{\infty} e^{-pt}\left[k - ak^2 - I^2\right]dt$$

$$\text{Sub to } \dot{k} = I - dk$$

$$k(0) = k_0$$

10

5. (a) Derive consumption possibility locus in the Leontief Static Open model

P.T.O.

(b) The input-output matrix is given by

$$A = \begin{pmatrix} 0.5 & 0.3 \\ 0.2 & 0.4 \end{pmatrix}$$

Direct labour requirements are

$a_{L1} = 0.6$ $a_{12} = 0.4$

Find the equation of the Consumption possibility locus. 5+5=10

6. State (derivation not required) and explain the economic implication of Hawkins-Simon condition. 10

7. (a) A person requires at least 10 units of A 12 units of B and 12 units of C – chemicals for his garden. A liquid product contains 5 units of A, 2 units of B and 1 unit of C per jar. A dry product contains 1 unit of A, 2 units of B and 4 units of C per carton. If the liquid product sells for Rs 3/- per jar and the dry product sells for Rs 2/- per carton, then formulate a Linear Programming problem with the objective of minimizing cost. 3+7=10

(b) In the following equations find the basic solution with x_3 as the non-basic variable

$$x_1 + 4x_2 - x_3 = 3$$

$$5x_1 + 2x_2 + 3x_3 = 3$$

8. (a) Consider the production problem, with fixed coefficient technology and given resources, formulated in the form of an LPP:

Maximize $Z = P_1X_1 + P_2X_2$

Subject to: $a_{11}X_1 + a_{12}X_2 \leq b_1$

$A_{21}X_1 + a_{22}X_2 \leq b_2$

$a_{31}X_1 + a_{32}X_2 \leq b_3$

$X_1, X_2, X_3 \geq 0$

Where P_i, X_i (i=1,2) are the market prices and quantities produced respectively. The resource endowments and technological coefficient are denoted by b_j and a_{ij} (j = 1,2,3) respectively.

(i) Formulate the dual problem

(ii) Interpret the meaning of the dual variables

(iii) In light of your interpretation of the dual variables, explain the meaning of the dual constraints. 3+2+3=8

(b) Using the LPP above, show that dual of a dual is the primal itself. 2