

B.A. (Honours) Examination, 2019
Semester–IV
Integrated Mathematics & Statistics
Course– S-1.4.5.P.6 (Subsidiary)

(For Back Candidates)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.

Answer *any four* questions.

1. Let $K(t)$ be the quantity of capital available in an industry at time t . Suppose $K(0)=500$ and the depreciation rate is 5% (0.05) and the investment rate is a constant 100 units.
 - (a) Solve for the expression showing the quantity of capital available as a function of time.
 - (b) Find the steady state value of the capital stock.
 - (c) Does $K(t)$ converge to this steady state? 7+1+2
2. Suppose demand and supply functions in a market are given by
$$q^D = 10 - 2p$$
$$q^S = 2 + 4p$$
Suppose that the speed of price change in the market is given by
$$\dot{p} = \frac{dp}{dt} = 0.2(q^D - q^S)$$
 - (a) Find the differential equation for price change in this market.
 - (b) If at $t=0$ the price is $p(0)=8$ find the solution to the differential equation.
 - (c) Does price converge to the steady state? 2+6+2
3. Find the solution to the non-autonomous linear first order differential equation
$$\dot{y} - y = e^{2t} \text{ and } y(0) = 1.$$
 10
4. Consider an economy where output per person $\left(y = \frac{Y}{L}\right)$ can be expressed as a concave function of the capital labour ratio $\left(k = \frac{K}{L}\right)$,

$$y = f(k) = k^{1/2}$$

The capital stock of the economy increases by the amount of investment which is by definition equal to the amount of output saved. Assume that the constant savings rate is 0.2 so that

$$\dot{k} = 0.2 Y$$

The labour force is assumed to grow at the constant rate $n = 0.1$

P.T.O.

(2)

- (a) Find the nonlinear differential equation that describes the growth of the economy in this model.
- (b) Find the steady state equilibria of this model if $y(0) = 0$.
- (c) Does the economy converge to the steady state? 5+2+3

5. Suppose that a fish population grows according to the function

$$g(y) = 2y(1 - y/2)$$

Where y is the stock of fish. Fish is harvested at a constant amount $3/4$. Will the fish population reach a steady state size if the initial population is $y_0 = 4/5$. What happens if the initial population was $y_0 = 1/5$. 7+3

6. Solve the following linear, second order differential equation

$$4\ddot{y} + 12\dot{y} - 7y = 28$$

$$y(0) = 2, \dot{y}(0) = 1. \quad 10$$

7. Suppose the demand and supply curves in a market is given by

$$q^d = 40 - Bp$$

$$q^s = 30 + 10p$$

The price adjustment equation in this model depends not only on the excess demand ($q^d - q^s$) but also on the inventories Accumulated from the past since $t = 0$, is

$$\ddot{p} = 0.25 (q^d - q^s) - 0.2 \int_0^t [q^s(r) - q^d(r)] dr$$

Find the value of B that ensures price converges to the steady state. 10

8. Solve the second order linear differential equation

$$\ddot{y} + 4\dot{y} + 5y = 10t^2$$

$$y(0) = 2$$

$$\dot{y}(0) = 1 \quad 10$$
