Five Year Integrated M.Sc. Examination, 2017

Semester-VIII

Course: MT-4-8-2B (Old)

(Mathematics: Numeric of Partial Differential Equations)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$. What is characteristics of a pde? Determine the characteristic curves for that pde. Discuss the significance of these results as domain of dependence and range of influence. 1+ 2+3+4=10

2. Develop the second order central approximation for $\frac{\partial^2 u}{\partial x^2}$ including the leading truncation error term. Derive the modified differential equation for the five-point approximation of the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ with $\Delta x=\Delta y$. Discuss consistency of this finite difference approximation. 3 + 5 + 2=10

3. What are the convergence and stability of a scheme? Write the FTCS scheme for the equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ at the point (ih, nk). Check the convergence of this scheme. 4+2+4=10

4. What is well-posed problem? State the Lax's equivalence theorem. Investigate the stability of the fully implicit difference equation:

$\frac{1}{k}(u_i^{n+1}-u_i^n)=\frac{1}{h^2}(u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1})$ approximating $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ at (ih, nk).

2+2+6=10

5. Deduce the following formulae and compute the corresponding order of accuracy: $\left.\frac{\partial u}{\partial x}\right|_i=\frac{-u_{i+2}+8u_{i+1}-8u_{i-1}+u_{i-2}}{12h}$; $\left.\frac{\partial^2 u}{\partial x\partial y}\right|_i=\frac{u_{i+1,j+1}-u_{i+1,j-1}-u_{i-1,j+1}+u_{i-1,j-1}}{4hk}$.

Hence find the order of accuracy of $\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial x\partial y}=0.$ 4+4+2=10

6. (a) What do you mean by wide and compact stencils? Deduce a higher order scheme for the equation $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0.$

(b) Derive the transformed form of the partial differential equation $\frac{\partial u}{\partial x}+\frac{\partial^2 u}{\partial x^2}=0$ given in physical plane to the computational plane. 5+5=10
