

Five Year Integrated M.Sc. Examination, 2017

Semester-VIII

Course: MT-4-8-1

(Mathematical Ecology)

Time: Four Hours

Full Marks: 80

Questions are of value as indicated in the margin.

[All the notations and symbols are used in their usual meaning]

Answer *any four* questions.

1. Describe elaborately which types of bifurcations are occurring for the following dynamical systems:

a) $\dot{x} = x^3 - \lambda x, \dot{y} = -y$

b) $\dot{x} = \lambda x - x^2, \dot{y} = -y$

c) $\dot{r} = r(\lambda - r^2), \dot{\theta} = -1$

[Here λ is the bifurcation parameter].

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2. What do you mean by basic reproduction number? Find the basic reproduction number from the following epidemic system:

$$\frac{dS}{dt} = A - \beta SI - dS + \sigma R$$

$$\frac{dI}{dt} = \beta SI - dI - mI$$

$$\frac{dR}{dt} = mI - (d + \sigma)R.$$

Also find the existence and stability criteria of all the equilibrium points. What happens if the numerical value of basic reproduction number becomes 1.

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3. a) Write down the solution procedure of a first order linear non-homogeneous difference equation. Also find the stability criteria of an equilibrium point of a first order difference equation. Hence find the equilibrium point(s) of the system $x_{n+1} = Kx_n / (b + x_n)$ and check the stability criteria of those points ($b, K > 0$).
- b) Find the Maximum sustainable yield (MSY) from the following single species harvest model:

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right) - dEN.$$

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4. Consider the following prey-predator model:

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K} \right) - axy$$

$$\frac{dy}{dt} = bxy - cy - dy^2.$$

Find all the equilibrium point(s) and their existence criteria. Also check the stability criteria of those points. Further examine whether there exists any limit cycle or not.

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5. a) Write a short note on 'Delay differential equation'.
- b) Write a short note on 'Optimal control theory'.

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P.T.O.

- c) What is an equilibrium point? What is the difference between local stability and global stability? What do you mean by local asymptotic stability? 5
- d) What do you mean by exponential growth and logistic type growth for a single species population system. What are the drawbacks of these two types of growth? Illustrate. 5
6. a) Solve the following optimal control problem:

$$\max_u \int_1^5 \left\{ u(t)x(t) - u(t)^2 - x(t)^2 \right\} dt \quad 10$$

Subject to $x'(t) = x(t) + u(t)$, $x(1) = 2$.

- b) What do you mean by a Lyapunov function? Where is this function used? With the choice of a suitable Lyapunov function, discuss the global asymptotic stability of the following prey-predator system at its interior equilibrium:

$$\begin{aligned} \frac{dx}{dt} &= rx \left(1 - \frac{x}{K} \right) - axy \\ \frac{dy}{dt} &= sy \left(1 - \frac{y}{L} \right) + bxy \end{aligned} \quad 10$$
