Five Year Integrated M.Sc. Examination 2017

Semester-VI

Course: PH-3-6-4

(Electromagnetic Theory)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin

Answer **Question No. 1** and any **three** from the rest

1. Answer any **five** from the following: $5 \times 2 = 10$
   (a) Prove that $A^\mu B_\mu$ is a Lorentz scalar.
   (b) Write down the transformation rule for a mixed tensor with covariant rank 3 and contravariant rank 2.
   (c) Define retarded potentials. Why it is different from ordinary potentials.
   (d) Write down the Lorentz transformation matrix for a Lorentz boost along y-axis with a velocity v.
   (e) Define 4-velocity and 4-acceleration.
   (f) Calculate the normal to a hyper-surface given by $\phi(x, y, z, t) = 3x^3 - 2ty + lnz + t^2$.
   (g) Prove that the metric tensor $g_{\mu\nu}$ transform as a rank two covariant tensor under a coordinate transformation of the form $x^i \to x'^i$

2. (a) Define covariant vector and contravariant vectors with examples.
   (b) Construct the rotation matrix for an arbitrary rotation about z axis. Now, using this transformation matrix show that covariant and contravariant vectors follow same transformation rules under ordinary rotation.
   (c) Define Lorentz scalar with at least one example. 4+4+2=10

3. (a) Calculate the Jacobian matrix for a Lorentz boost along $+ve$ x- direction. Hence, show that the four dimensional volume element $(dx^0 dx^1 dx^2 dx^3)$ remains invariant under Lorentz transformation.
   (b) Prove that the charge density (ρ) transforms as the zeroth component of a four vector.
   (c) Define 4-current.
   (d) Show that the continuity equation for electric charge remains form invariant under Lorentz transformation. 2+3+2+3=10

4. (a) Define 4-potential of an Electromagnetic field.
   (b) Using the 4-potential define field strength tensor $(F_{\mu\nu})$ and obtain the components of the same.
   (c) Show that $F_{\mu\nu}$ is a guage invariant quantity. 2+5+3=10

P.T.O.

5. (a) Consider a point charge (q) is at rest at the origin in an inertial frame S'. Calculate the electric field and magnetic field due to the same charge as measured by an observed standing on the inertial frame S, which is moving to the left at speed v relative to S'. Here you can use the following transformation rules

$$\begin{aligned} E_1' &= E_1, E_2' = \gamma(E_2 - \beta B_3), E_3' = \gamma(E_3 + \beta B_2) \\ B_1' &= B_1, B_2' = \gamma(B_2 + \beta E_3), B_3' = \gamma(B_3 - \beta E_2) \end{aligned}$$

(b) Show that $\vec{E}.\vec{B}$ is a Lorentz scalar. 6+4=10

6. (a) Maxwell's equations with source terms can be expressed as $\partial_\mu F^{\mu\nu} = \mu_0 J^\nu$. Starting from this equations prove continuity equation.

(b) Electric and magnetic field due to a moving point charge q is given by

$$\begin{aligned} \vec{E}(\vec{r}, t) &= \frac{q}{4\pi\epsilon_0} \frac{d}{(\vec{d}.\vec{u})^3}[(c^2 - v^2)\vec{u} + \vec{d} \times (\vec{u} \times \vec{a})], \\ \vec{B}(\vec{r}, t) &= \frac{1}{c}[\hat{d} \times \vec{E}(\vec{r}, t)], \end{aligned}$$

where, $u = c\hat{d} - \vec{v}$, $d = c(t - t_r)$ and $\vec{a}$ is the acceleration of the charged particle.

i. Identify the radiation part of the electric and magnetic field. Why these are called radiation part?

ii. Hence calculate the Poynting vector for the radiation fields w.r.t a frame in which the charge was instantaneously at rest at retarded time (t_r).

3+(3+4)=10