Five Year Integrated M.Sc. Examination 2017

Semester-VI

Course: PH-3-6-2

(Atomic Physics and Laser)

Time: Four Hours **Full Marks: 80**

Questions are of value as indicated in the margin
Answer **Question No. 1** and any **four** from the rest

1. Answer any **five** from the following: $5 \times 4 = 20$

   (a) Briefly explain the Lamb -Retherford experiment to determine the energy gap between $2s_{1/2}$ and $2p_{3/2}$ states of Hydrogen atom. Draw a schematic diagram to explain the experimental set-up.

   (b) Write down the two particle wave functions for (i) spinless Boson and (ii) spinless Fermions. Using the Fermionic two particles state, obtain Pauli Exclusion Principle. Write down the N particle state for spinless Fermions?

   (c) Define life time of an excited state. Show that for uniform magnetic field ($\vec{B}$) one can express the magnetic vector potential as $\vec{A} = \frac{1}{2}(\vec{B} \times \vec{r})$.

   (d) What do you mean by Autoionization ? Why does the spin part of the ground state wave function of Helium atom is antisymmetric ?

   (e) What do you mean by fine structure in ESR spectra? Explain briefly.

   (f) (i) What is used as a standard reference for studying the proton NMR of an unknown sample and why? (ii) What are the advantages of MRI over CT scan or traditional X-rays in clinical imaging?

   (g) Explain briefly the working principle of a ruby laser.

2. (a) What do you mean by parity symmetry? Define parity operation in spherical polar coordinate. Hence calculate the eigenvalue of the parity operator for Hydrogen atom wave function $\langle r\theta\psi|nlm\rangle = \psi_{nlm}(r, \theta, \phi)$. Explicit form of the Hydrogen atom wave function is given below

$$\psi_{nlm}(r, \theta, \phi) = C_{nlm}\, R_{nl}(r)\, e^{im\phi} P_l^m(\cos\theta),$$

   where,

$$P_l^m(x) = (1 - x^2)^{|m|/2} \left(\frac{d}{dx}\right)^{|m|} P_l(x),$$

   and C_{nlm} is a constant term independent of coordinates.

   (b) Using parity symmetry show that $\langle nlm|\mathbf{r}|nlm\rangle$ is zero, where $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

   (c) Calculate the commutator between L_z and x, y and z. Hence show that $\langle n'l'm'|\mathbf{r}|nlm\rangle$ is zero unless $m = m'$ or $m - m' = \pm 1$.

   (d) Using $[L^2[L^2, \mathbf{r}]\,] = 2\hbar^2(\mathbf{r}L^2 + L^2\mathbf{r})$, prove that $\langle n'l'm'|\mathbf{r}|nlm\rangle$ is zero unless $l = l'$.

(1+2+2)+2+(3+2)+3=15

3. (a) Suppose a quantum mechanical system is described the Hamiltonian $H = H_0 + V$, where $H_0 >> V$. If $|\phi_n\rangle$ is the n-th eigenfunction of the unperturbed Hamiltonian (H_0) with eigenvalue E_n, then show that the first order correction to the energy eigenvalue is given by $E_n^1 = \langle \phi_n | V | \phi_n \rangle$.

(b) Show that the relativistic correction for the Hydrogen atom is given by

$$H_1' = -\frac{p^4}{8m_e^3 c^2} ,$$

where p is the momentum of the electron and m_e is the electronic mass.

(c) Using first order perturbation theory calculate the energy correction due to H_1'. You can use following relations without proof

$$\begin{aligned} \left\langle \frac{1}{r} \right\rangle_{nlm} &= \frac{Z}{an^2} , \\ \left\langle \frac{1}{r^2} \right\rangle_{nlm} &= \frac{Z^2}{a^2 n^3 (l + 1/2)} . \end{aligned}$$

6+4+5=15

4. (a) Briefly discuss the difference between normal Zeeman effect and anomalous Zeeman effect.

(b) Suppose the unperturbed Hamiltonian of Hydrogen atom with L-S coupling is given by

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{(4\pi\epsilon_0)r} + \xi(r)\,\mathbf{L}.\mathbf{S} ,$$

Identify the appropriate eigenfunction basis for the aforesaid Hamiltonian.

(c) Write down the perturbative Hamiltonian if the the atom is placed in a weak magnetic field B directed along z-direction.

(d) Hence show that the energy shift due to the magnetic field is given by

$$\Delta E_{m_j} = g\mu_B B m_j ,$$

where, g is the Lande g factor, and $\mu_B = \frac{e\hbar}{2m_e}$ is Bohr magneton.

(e) Draw the diagram depicting the electric dipole transitions between the $n = 1$ and $n = 2$ levels of Hydrogen, in a weak magnetic field.

4+1+1+5+4=15

5. (a) Show the diagrams for Larmor precessions of the magnetic moments and angular momenta associated with the spin of an electron (both spin up and down states) in the presence of an external magnetic field along Z-axis.

   (b) Explain hyperfine structure in the ESR spectra of methyl radical. Determine the number of peaks and their relative intensities showing the family tree of couplings which produce the peaks.

   (c) Calculate the Larmor frequency associated with the spin of an electron at an external magnetic field of $0.34\,T$. Given, Bohr magneton $= 9.27\times10^{-24}\,JT^{-1}$, $h = 6.62\times10^{-34}\,Js$.

4+(4+4)+3=15

6. (a) Starting from the interaction of a nucleus of spin I (no specific value) in presence of an external magnetic field B_z, calculate the separation between neighbouring spin energy levels. Also, calculate the Larmor frequency associated with the nucleus.

   (b) A particular NMR instrument operates at $30.256\,MHz$. Determine the magnetic fields necessary to bring ^1H and ^{13}C nuclei to resonate at that frequency. Given, $\beta_n = 5.05\times10^{-27}\,JT^{-1}$, $g_H = 5.585$, $g_C = 1.404$, $h = 6.62\times10^{-34}\,Js$.

   (c) Explain the concept of chemical shift in NMR spectroscopy. (3+2)+4+6=15

7. (a) What are the basic characteristics of laser? Explain briefly.

   (b) Explain the basic principle of working of He-Ne laser. What is the role of He gas in a He-Ne laser?

   (c) In a $0.2\,m$ long Nd:YAG laser, gain bandwidth is a Lorentzian shape with a FWHM of $0.5\,nm$ and the peak is at $1064\,nm$. The refractive index of the gain medium at $1064\,nm$ is 1.82. Calculate the spacing between two adjacent longitudinal modes. Also, calculate the longitudinal mode number for the central frequency and total number of longitudinal modes present.

   (d) What are the advantages of gas lasers over solid state lasers? 3+(5+1)+4+2=15