

Five Year Integrated M.Sc. Examination, 2017

Semester-VI

Course: MT-3-6-4 (Old) (Mathematical Modelling)

Time: Three Hours

Full Marks: 40

Questions are of value as indicated in the margin.

[All the notations and symbols have their usual meaning]

Answer *any four* questions.

1. Answer *any five* questions: 5x2
 - a) Write down the Postulates of Poisson process.
 - b) What is the difference between local asymptotic stability and global asymptotic stability?
 - c) Derive the local stability condition at a fixed point of a first order difference equation of the form $x_{n+1} = f(x_n)$.
 - d) If $x = 5\sin t - 4\cos t$, then find the amplitude and period of oscillation.
 - e) Solve the following problem by graphical method:
Maximize $z = 2x + y$
Subject to $x + 4y \leq 8$
 $x, y \geq 0$.
 - f) What is the necessity of Mathematical modelling?
2. Derive the solution procedure of a second order linear homogeneous difference equation. Use this method to find n-th member of Fibonacci sequence. Give at least one real world example where Fibonacci sequence can be used. 10
3. Derive the differential equation for pure birth process. Solve this system for Yule-Furry process. Also derive the mean and variance of this method. 10
4.
 - a) Suppose a particle of mass 'm' is projected from the origin with velocity V at an angle θ with the horizon. Find the range and time of flight. 5
 - b) Consider the following spring mass system:
$$m \frac{d^2x}{dt^2} = -kx + \beta x^3, \beta > 0.$$

What are the equilibrium points of the above system? Also discuss the nature of those equilibrium points. 3
- c) What is the Jacobian matrix? 2
5. Consider the following biological model:
$$\frac{dS}{dt} = A - \beta SI - dS + mI$$
$$\frac{dI}{dt} = \beta SI - dI - mI.$$

Find all the possible equilibrium points of above system. Discuss their feasibility criteria and stability criteria. 10
6. Derive the optimal economic order quantity (Q), minimum cost (C) and optimum time of an inventory model with no shortages and instantaneous production. Hence solve the following question:
A factory consumes 5000 units of components per year. The ordering and receiving costs are Rs. 300 per order while the trucking cost is Rs. 1200 per order. Also interest cost is Rs. 0.06 per unit per year, Deterioration cost Rs. 0.004 per unit per year and storage cost Rs. 1000/- per year for 5000 units. Calculate the optimal economic order quantity, minimum average cost and optimum time. 6+4