Five Year Integrated M.Sc. Examination, 2017

Semester - IV

Course: PH-2-4-2

(Statistical Mechanics)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin

Answer **Question No. 1** and any **three** from the rest

1. Answer any **five** from the following: $5 \times 2 = 10$

   (a) Define phase space? How do you represent a microstate in phase space?

   (b) Draw the phase space diagram of a particle with energy E, confined within a one dimensional line extended from $-L$ to L.

   (c) Define Density of states.

   (d) Using the definition of canonical partition function show that the entropy is an additive quantity for weakly interacting systems.

   (e) Write down the two particle wave-function for Fermions. Hence show that the two-particle wave function vanishes if both particles occupy same position.

   (f) Why do the quantum identical particles indistinguishable?

   (g) Briefly explain Gibbs' paradox with an example.

2. (a) Obtain the canonical probability distribution function for a system which is in thermal equilibrium with a reservoir at a temperature $T\,K$.

   (b) Obtain the expression of average energy (E), pressure (p) and entropy (S) for the canonical ensemble. 4+(2+2+2)=10

3. (a) State the postulate of equal a priori probabilities. Is it valid for all types of ensembles?

   (b) How does the total number of microstates for a isolated system behave at equilibrium? Define reversible and irreversible processes with examples.

   (c) Can you draw the phase space trajectory of a quantum system? Give justification in support of your answer.

   (d) Define grand canonical ensemble. Write the expression for corresponding partition function. (2+1)+(1+2)+2+(1+1)=10

4. (a) Calculate the canonical partition function of an classical ideal gas consists of N number of gas particles confined with a volume V. Hence calculate the entropy and average energy of the gas.

   (b) Derive the expression for thermal wave length ($\bar{\lambda}$). How is it related with inter particle distance (R) for a quantum system. (3+2+1)+(3+1)=10

5. (a) Consider an isolated system in equilibrium. Establish the relation between the temperatures of two different parts of this system.

   (b) Consider an isolated system of N non-interacting particles, each fixed in position. Each particle may exist in one of the two states $E = 0$ or $E = \epsilon$. Treat the particles as distinguishable.

      i. Calculate the number of accessible states, $\Omega(E, N)$ for this system.

      ii. Express entropy S, in terms of $\Omega(E, N)$. Apply Starling approximation to simplify this result. ($lnN! = NlnN - N,$ for $N \to \infty$)

      iii. Calculate the maximum value of the entropy. 4+(2+2+2)=10

6. (a) Consider a microcanonical ensemble of a system of N non-interacting Bosons confined to a volume V and sharing a total energy E. Let us assume that the energy spectrum is discrete in which the i-th level corresponds to the energy ϵ_i with degeneracy factor g_i. If n_i is the average number of particles in i-th level, then calculate the number of distinct microstates corresponding to the set of occupation numbers $\{n_i\}$.

   (b) Six (6) particles are to be distributed in ten (10) degenerate energy levels with energy ϵ. Calculate the number of ways of distributing these particles if they are (a)classical identical particles, (b)Fermions and (c)Bosons.

4+(2+2+2)=10