# Five Year Integrated M.Sc. Examination, 2017

**Semester - IV**

**Course: PH-2-4-1**

**( Quantum Mechanics )**

**Time: Three Hours** **Full Marks: 40**

Questions are of value as indicated in the margin.
Answer ***Question No.1*** and ***any three*** from the rest.

1. Answer ***any four*** of the following: 2.5x4=10
   a) He at saturated vapor pressure remains liquid down to $T = 0$. Explain.
   b) Show that the ground state of harmonic oscillator is a minimum uncertainty wave packet state.
   c) Consider a one-dimensional negatively charged harmonic oscillator in electric field $E$. Write down its Hamiltonian and find its eigenfunctions.
   d) Let $|0\rangle$ be the ground state of a one-dimensional harmonic oscillator. Calculate $\langle 0|e^{ikx}|0\rangle$.
   e) Show that $e^{ip\lambda/\hbar} x e^{-ip\lambda/\hbar} = x + \lambda$. Here $\lambda$ is a c-number.
   f) $B = e^{i\lambda G} A e^{-i\lambda G}$. $\lambda$ is a c-number. $G$ and $A$ are operators. Find first there terms of Baker-Hausdorff formula.

2. a) Calculate the density of modes for standing waves of electromagnetic radiation in a cavity. 4
   b) Let $u_\nu$ be the radiated energy density from a blackbody at a frequency $\nu$. Derive Rayleigh-Jeans formula that connects $u_\nu$ with the temperature of the blackbody. 2
   c) Wilhelm Wien showed that on thermodynamic ground, the radiation energy density $u_\nu$ must have the form, $\nu^3 f\left(\nu/T\right)$, where $f$ is a function.
   Derive Planck's formula for $u_\nu$ and show that it has the Wien's form. 4

3. a) Define photoelectric effect. 1
   b) What would be the experimental findings on the basis of wave theory of light? Describe any two expected results. 2
   c) State Einstein's theory to explain photoelectric effect. 1
   d) Using photoelectric effect experiment, derive Planck's constant, $h$ and work function $W$. 3
   e) Radiation of wavelength $\lambda = 290$ nm falls on a metal surface for which work function, $W$ is 4.05 ev. What potential is needed to stop the most energetic electron? 3

4. a) State de Broglie hypothesis. 1
   b) Using the above hypothesis calculate the energy levels for a particle in a one-dimensional potential box of length, $L$. 2
   c) Using de Broglie hypothesis, calculate the energy levels of hydrogen like atoms. 4
   d) What is the de Broglie wavelength of an electron of energy 200 Mev.
   $m_e = 0.51$ Mev/c$^2$.
   $h = 6.63 \times 10^{-34}$ J-S
   $c = 3 \times 10^8$ m/sec. 3
   $1\text{ev} = 1.602 \times 10^{-19}$ J.

**P.T.O.**

5. a) Consider the diffraction of a beam of thermalized He atom by a simple cubic lattice $(d \simeq 0.20 \text{ nm})$, where $d$ is the lattice spacing. At what temperature for He atoms would diffraction be appreciable?

$$m(He) \simeq 4.002 \times 10^{-3} \text{ kg/mole}$$

$$N_0 = \text{ Avogrado number } \simeq 6.02 \times 10^{+23} \big/ \text{mole}$$ 4

$$k_B = \text{ Boltzmann's constant} \simeq 1.38 \times 10^{-23} \text{ J/k.}$$

b) A particle of mass $m$ is bound in the delta function potential well, $V(x) = -|V_0|\delta(x)$. What is the probability that a measurement of its momentum would yield a value greater than $p_0 = \frac{m|V_0|}{\hbar}$? 4

c) Consider a potential

$$V(x) = -\frac{\hbar^2 a^2}{m}\text{sech}^2(ax),$$

where "$a$" is a positive constant and "sech" stands for the hyperbolic secant.
For this one-dimensional quantum system, the ground state wavefunction,

$$\psi_0(x) = A \text{ sech}(ax).$$

Find the energy of the ground state. 2

6. Consider an one-dimensional quantum system with potential $V(x)$ given by

$$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2, \qquad x > 0,$$

$$= \infty \qquad x \leq 0.$$

a) Find its unnormalized eigenfunctions. 2

b) Find its energy eigenvalues. 2

c) Normalize its ground state eigenfunction. (Note: $H_1(x) = 2x$.) 2

d) Find the position-momentum uncertainty, i.e. $\Delta x \, \Delta p$ value in its ground state. 4