Use seperate answer script for each unit

B.A./B.Sc. (Honours) Examination 2019

Semester – VI

Mathematics

Course: BMC-64

(Mechanics-IV & Mathematical Modelling)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Marks: 24)

Mechanics-IV (Hydrostatics)

Answer ***any four*** questions

1. a) Define stress, thrust and shearing stress on a surface of a fluid. 1.5
   b) Show that in a homogeneous fluid at rest under gravity, the difference of the pressures between any two points in the same vertical line is proportional to the difference of their depths. 2
   c) A small uniform circular tube, whose plane is vertical, contains equal quantities of fluids whose densities are ρ and $\sigma, (\rho > \sigma)$, and do not mix. If they together fill half the tube, show that the radius passing through the common surface makes with the vertical an angle $\tan^{-1}\frac{\rho-\sigma}{\rho+\sigma}$. 2.5

2. a) Obtain a necessary condition for a fluid of mean density ρ to be in equilibrium under the external force $\boldsymbol{F} = (X, Y, Z)$ per unit mass of the fluid. 3
   b) Show that if the field of force is conservative, the surface of equi-pressure, equi-density and equi-potential energy coincide. Hence find the differential equations of the equi-density and equi-pressure surfaces. 1.5+1.5

3. a) A given volume V of liquid is acted upon by forces $-\mu x/a^2, -\mu y/b^2$ and $-\mu z/c^2$ along the rectangular Cartesian coordinate axes, where μ, a, b and c are constants. Find the equation of the free surface. 2
   b) Show that the pressure at a small depth z below the surface of a sphere of water attracted to the centre of the sphere with a force producing an acceleration μ/r^2 at a distance r is approximately $\prod + g\rho\left(z + z^2/a\right)$, where a is the radius of the sphere, $\prod$ the pressure at the free surface, and g the attraction of a unit mass at the surface of the sphere. 4

4. a) Explain the notion of thrust of a heavy homogeneous liquid on a plane surface. 2
   b) A hollow weightless cone of vertical angle 2α is filled with fluid, and suspended freely from a point on the rim of its base. Show that the ratio of the thrust on the base and the weight of fluid, the cone would contain, is $12\sin^2\alpha : \cos\alpha\sqrt{1+15\sin^2\alpha}$. 4

P.T.O.

5. a) Define the centre fo pressure of a given area in a fluid. Find the depth of the centre of pressure of a plane area immersed in a fluid under the action of gravity. 1+2
   b) A lamina in the shape of a quadrilateral *ABCD* has its side *CD* in the surface of a liquid and the sides *AD*, *BC* vertical and equal to α, β respectively. Show that the depth of the centre of pressure is $\frac{1}{2}\frac{\left(\alpha^2+\beta^2\right)\left(\alpha+\beta\right)}{\alpha^2+\alpha\beta+\beta^2}$. 3

6. a) State the conditions of equilibrium of a freely floating body immersed wholly or partially in a fluid. 2
   b) Define Metacentre of a freely floating body, and hence obtain the condition(s) for stable and unstable equilibria. What is surface of buoyancy? 2+1+1

Unit-II (Marks: 20)

Mathematical Modelling

Answer ***any two*** questions

1. a) State the concept of mathematical modelling. Sketch a schematic reprsentation of the modelling process. Classify the models and indicate their differences with illustrations. 4
   b) Show that the energy is not conserved in the spring-mass system in the presence of friction unlike that in frictionless system. 4

2. a) Consider $\dot{x}=f\left(x\right)$ as one-dimensional autonomous dynamical system with $f:\mathbb{R}\to\mathbb{R}$ which is strongly differentiable on $\mathbb{R}$. If $x_0\in\mathbb{R}$ is a fixed point with $f\left(x_0\right)=0$, then show that x_0 is asymptotically stable if $f'\left(x_0\right)<0$ and unstable if $f'\left(x_0\right)>0$. 4
   b) Find the fixed points of the following systems and examine their stability
      (i) $\frac{dx}{dt}=4x^2\left(x+1\right)^3\left(x-3\right)^2$
      (ii) $\frac{dx}{dt}=x^2-x^5$ 4

3. a) Define Lyapunov stability of the solutions of the *n*-dimensional dynamical system $\dot{x}=X\left(x,t\right)$. Hence show that all solutions of the *n*-dimensional system $\dot{x}=-x$ are stable for $t\geq 0$ in the Lyapunov sense. 4
   b) Evaluate the Jacobian for the following autonomous system
      $\dot{x}_1=x_1^2\left(x_1-x_2\right)^3$
      $\dot{x}_2=x_2^3\left(x_1-x_2\right)^2$
      and hence examine the stability of the system at $\left(0,\,0\right)$. 4

———