Use separate answer script for each Unit

B.A./B.Sc. (Honours) Examination 2019

Semester – VI

Mathematics

Course: BMC-63

(Tensor Calculus & Mathematical Statistics)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Marks: 20)

Tensor Calculus

Answer ***any two*** questions

1. a) Define covarient tensor. A covarient tensor has components $xy, 2y-z^3, xz$ in rectangular coordinate. Find its covarient component in spherical coordinates. 3

b) Show that the expression $A(i,j,k)$ is a tensor if its inner product with an arbitrary tensor B_k^{jl} is a tensor. 3

c) Let $a_{ij}\ (i,j=1,2,...,n)$ be n^2 quantities in a certain coordinate system. If A^i and B^j are two arbitrary tensors of type $(1,0)$ such that $a_{ij}A^iB^j$ is an invarient, then a_{ij} are the components of a tensor of type $(0,2)$. 2

d) If the relation $A_{ijkl}u^iv^ju^kv^l=0$ holds for arbitrary contravarient vectors u^i and v^i, prove that $A_{ijkl}+A_{ilkj}+A_{kjil}+A_{klij}=0$. 2

2. a) If the metric in rectangular Cartesian coordinates is given by $ds^2=(dx^1)^2+(dx^2)^2+(dx^3)^2$, then find the metric in cylindrical polar coordinates. 3

b) Prove that

$$\lambda^r,r=\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^i}\left\{\sqrt{g}\lambda^i\right\},$$

where $g=\left|g_{ij}\right|\neq 0,\ g_{ij}$ being the fundamental metric tensor. 4

c) Show that the Kronecker delta is a mixed tensor of order 2. 1

d) If $a^{ij}u_iu_j$ is an invarient for an arbitrary covarient vector u_i, show that $a^{ij}+a^{ji}$ is a tensor. 2

3. a) If a vector has components $\dot{x},\dot{y}$ in rectangular Cartesian coordinate then show that $\dot{r},\dot{\theta}$ are components in polar coordinates and if a vector has components $\ddot{x},\ddot{y}$ in Cartesian coordinates then prove that its component in polar coordinates are $\ddot{r}-r\dot{\theta}^2,\ \ddot{\theta}+\frac{2}{r}\dot{r}\dot{\theta}$, where dots indicates differentiation with respect to a parameter *t*. 3

b) Find the conjugate matric tensor in a Reimanian space V_3 in which the distance ds is given by

$$ds^2=5(dx^1)^2+3(dx^2)^2+4(dx^3)^2-3(dx^1)(dx^2)-3(dx^2)(dx^1)+2(dx^2)(dx^3)+2(dx^3)(dx^2)$$ 3

P.T.O.

3. c) Calculate the Christoffel symbols $\begin{Bmatrix} 1 \\ 22 \end{Bmatrix}, \begin{Bmatrix} 2 \\ 21 \end{Bmatrix}, \begin{Bmatrix} 1 \\ 33 \end{Bmatrix}, \begin{Bmatrix} 2 \\ 33 \end{Bmatrix}, \begin{Bmatrix} 3 \\ 31 \end{Bmatrix}, \begin{Bmatrix} 3 \\ 32 \end{Bmatrix}$ corresponding to the metric $ds^2 = \left(dx^1\right)^2 + \left(x^1\right)^2 \left(dx^2\right)^2 + \left(x^1 \sin x^2\right)^2 \left(dx^3\right)^2$. 4

Unit-II (Marks: 20)
Mathematical Statistics
Answer ***any two*** questions

1. a) Explain the terms: Population, Random sample and Sample values. Show that the distribution of a sample is the statistical image of that of the population. 3+2
   b) Define sampling distribution of a statistic. When a statistic is said to be consistent and unbiased estimate of a population parameter? 1+(1+1)
   c) Prove that for a sample size n from a normal (m, 1) population, the arithmetic mean is an unbiased and consistent estimate of the population mean. 2

2. a) Explain the method of maximum likelihood for estimation of population parameters of a continuous random variable X. Use this method to find an estimate of p for the binomial (N, p) population. 2+3
   b) Define confidence interval and confidence coefficient of a population parameter α. 2
   c) The marks obtained by 17 candidates in an examination have a mean 57 and variance 64. Find a 99% confidence interval for the mean of the population, assuming it to be normal. Given that $P\left(t > 2.92\right) = 0.005$. 3

3. a) Define null and alternative hypotheses, and two types of errors associated with the test of a statistical hypothesis. When a critical region for testing a simple hypothesis against an alternative one is said to be the best critical region? 4+1
   b) Use Neyman-Pearson theorem to construct a test for a normal $\left(m, \sigma\right)$ population to find the best critical region. 5

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