Use separate answer script for each unit

B.A./B.Sc. (Honours) Examination 2019

Semester – VI

Mathematics

Course: BMC-61

(Analysis-VI)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Marks: 20)

Complex Analysis

Answer ***any two*** questions

1. a) Show that $\lim\limits_{z\to a}\dfrac{\overline{z-a}}{z-a}$ does not exist.

   b) If $f(z)$ is continuous at $z=a$ and $f(a)\neq 0$ then show that $\dfrac{1}{f(z)}$ is also continuou at $z=a$.

   c) Show that the function $f(z)$, defined by

$$f(z)=\frac{x^3y}{x^6+y^2},\ z\neq 0$$
$$=\ 0 \qquad ,\ z=0$$

   is not continuous at the origin.

   d) If $f(z)$ is differentiable at a, then prove that it must be continuous at a. Give an example to show that the converse is not necessarily true in general. 2+3+2+3

2. a) Deduce Cauchy-Reimann equations for a function to be differentiable at a point in $\mathbb{C}$.

   b) Show that the function $e^x\cos y$ is harmonic and find its conjugate harmonic function.

   c) State Cauchy's general principle of convergence of a sequence of complex numbers.

   d) Prove that every absolutely convergent series is convergent. Is the converse true? Justify your answer. 4+2+1+3

3. a) Prove that a series $\sum\limits_{n=1}^{\infty} z_n$ of complex terms is absolutely convergent if $\limsup\limits_{n\to\infty}\left|\dfrac{z_{n+1}}{z_n}\right|<1$.

   b) If α and β be two fixed points of a bilinear transformation $w=\dfrac{a+bz}{c+dz}$, then show that the transformation can be written as $\dfrac{w-\alpha}{w-\beta}=k\dfrac{z-\alpha}{z-\beta}$, where $\left(\sqrt{k}+\dfrac{1}{\sqrt{k}}\right)^2=\dfrac{(b+c)^2}{bc-ad}$.

P.T.O.

3. c) State Orientation Principle.
   d) Find the stereographic projection of a circle in the complex plane whose equation is given as $x^2+y^2+2gx+2fy+c=0$. 3+3+1+3

Unit-II (Marks: 20)
Metric Spaces
Answer ***any two*** questions

1. a) Define a 'Discrete Metric Space'.
   b) If (X,d) is a metric space and $d_1 : X\times X \to \mathbb{R}$ be such that $d_1(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$, then show that (X,d_1) is also a metric space.
   c) Show that the set of all sequences of real numbers is a metric space with respect to a metric to be defined by you.
   d) If A and B are two non-empty subsets of a metric space X, then show that $\overline{A\cup B}=\overline{A}\cup\overline{B}$.
   e) Show that the closure of a set A consisting of a single point in a metric space X is the set A itself. 1+2+3+2+2

2. a) Define a closed sphere in a metric space (X,ρ). Show that a closed sphere in a metric space X is also a closed set.
   b) Let $A\subset X$, where X is a metric space. Show that A is open if and only if for every point $x\in A$, there exists a neighbourhood S of x such that $S\subset A$.
   c) Let G be a non-empty subset in a metric space (X,d). Then, show that G is open if and only if it is a union of open spheres in X.
   d) Let X be a metric space. Show that a set $A\subset X$ is open if and only if $A=Int(A)$.
   e) Let $A\subset X$, where X is a metric space. Show that A is closed iff $Br(A)\subset A$, where $Br(A)$ denotes the boundary of A. 3+2+2+2+1

3. a) Let X be the set of all polynomials $P(t)$, $0\le t\le 1$ with real coefficients is in the metric d defined by $d(P,Q)=\underset{0\le t\le 1}{Sup}\left|P(t)-Q(t)\right|$, where $P(t),\ Q(t)\in X$. Examine if (X,d) is an incomplete metric space.
   b) Let (X,ρ) be a metric space. Show that $\delta(\overline{S})=\delta(S)$, where $S\subset X$ and $\delta(S)$ denotes the diameter of S.
   c) State and prove Cantor's intersection theorem.
   d) Show that any contraction mapping in a metric space (X,ρ) is uniformly continuous. 2+2+5+1

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