For Non-CBCS Students only

B.A./B.Sc. (Honours) Examination 2019

Semester – IV

Mathematics

Course: BMC-41

(Analysis-IV)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.
Answer ***any four*** questions. **4×10=40**

1. a) Evaluate the integral $\int_0^1 dx \int_x^{\sqrt{x}} \frac{y\,dy}{(1+xy)^2(1+y^2)}$ by changing the order of the integration. 5

   b) Evaluate $\iiint_V \frac{dx\,dy\,dz}{(1+x+y+z)^3}$, where V is the tetrahedron bounded by the planes $x=0,\ y=0,\ z=0$ and $x+y+z=1$. 5

2. a) Evaluate $\iint_E \frac{dx\,dy}{\sqrt{(x+y+1)^2-4xy}}$, by using the transformation $x=u(1+v)$, $y=v(1+u)$, where E is the triangle with vertices $(0,0)$, $(2,0)$ and $(2,2)$. 5

   b) Test the convergence of the integral $\int_0^\infty \frac{\cos x\,dx}{1+x^2}$. 2

   c) Discuss the convergence of the Beta function $\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$. 3

3. a) Evaluate : $\int_0^1 \sqrt{1-x^4}\,dx$. 2.5

   b) Evaluate : $\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}}\,dx \int_0^{\pi/2} \sqrt{\sin x}\,dx$. 2.5

   c) Using differentiation under integral sign, find the value of $F(2,3)$, where $F(a,b)=\int_0^{\pi/2} \frac{dx}{(a^2\cos^2 x+b^2\sin^2 x)^2}$. 5

4. a) Show that the function $f:[0,1]\to\mathbb{R}$ defined by $f(x)=\begin{cases} x^2\sin\frac{1}{x}; & \text{if } x\neq 0 \\ 0\ ; & \text{if } x=0 \end{cases}$, is a function of bounded variation on $[0,1]$. 5

   b) Let f is a monotonic function on $[a,b]$, and α is a monotonic increasing function on $[a,b]$. If α is continuous then show that f is Riemann-Stieltjes integrable on $[a,b]$. 5

P.T.O.

5. a) Let $f:[a,b]\to\mathbb{R}$ be a bounded function and P be a partition of $[a,b]$ with $\|P\|=\delta$. If P_k be a refinement of P with k additional points, then show that $U(P,f)\geq U(P_k,f)$, $L(P,f)\leq L(P_k,f)$; $U(P_k,f)-U(P,f)\geq(m-M)kS$, $L(P_k,f)-L(P,f)\leq(M-m)k\delta$, where $m=\inf_{x\in[a,b]} f(x)$ and $M=\sup_{x\in[a,b]} f(x)$. 2+3

   b) Find the upper and lower sum of $f:[0,1]\to\mathbb{R}$ defined by

$$f(x)=\begin{cases} x\ ; & \text{if } x \text{ is rational} \\ x^2\ ; & \text{if } x \text{ is irrational} \end{cases}$$

5

6. a) Show that if $f(x)$ be monotone on $[a,b]$, then $\int_a^b f(x)$ lies in between $f(a)(b-a)$ and $f(b)(b-a)$. 3

   b) Let $f(x)$ be Riemann integrable and non negative on $[a,b]$. If $f(x)$ be continuous at c on $[a,b]$ and $f(c)>0$, then prove that $\int_a^b f(x)\,dx>0$. 3

   c) Find $\lim_{x\to 0}\dfrac{\int_0^x \sin t\,dt}{x^2}$, if it exists. 2

   d) Let $f(x)=\begin{cases} x\ ; & \text{if } x\neq\frac{1}{n} \\ 1\ ; & \text{if } x=\frac{1}{n};\ n=1,2,\ldots \end{cases}$

   Is f Riemann integrable on $[0,1]$? 2

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