Use Separate Answer Scripts for each Unit

B.A./B.Sc. (Honours) Examination 2019

Semester – IV

Mathematics

Allied Course: BMA-41

(Linear Programming Problem and Numerical Methods)

Time: Four Hours **Full Marks: 60**

Questions are of values as indicated in the margin.

Notations and symbols have their usual meanings.

UNIT-I (Marks=32)

(Linear Programming Problem)

Answer ***question no. 1*** and ***three*** from the rest

1. Answer any two questions: $2\times1=2$

 a) Is the set of vectors $\{(1,2,3),\ (1,1,1),\ (0,0,0)\}$ linearly independent? Justify your answer.

 b) Define feasible solution in an L.P.P.

 c) Let X be the set of points lying on the boundary of a circle. Find the convex hull of the set X.

2. a) A hospital has the following minimum requirements for nurses.

Period	Clock time (24 hours day)	Minimum number of nurses required
1	6 a.m. - 10 a.m.	60
2	10 a.m. - 2 p.m.	70
3	2 p.m. - 6 p.m.	60
4	6 p.m. - 10 p.m.	50
5	10 p.m. - 2 a.m.	20
6	2 a.m. - 6 a.m.	30

 Nurses report to the hospital ward at the beginning of each period and work for eight consecutive hours. The hospital authority wants to determine the minimum number of nurses so that there may be sufficient number of nurses available for each period. Formulate this problem as an L.P.P. 5

 b) Solve the following L.P.P. graphically:

 Maximize, $z = 3x_1 + 2x_2$

 Subject to $x_1 - x_2 \le 1$ 5

 $x_1 + x_2 \ge 3$

 $x_1, x_2 \ge 0.$

3. a) Rewrite the following minimization problem to a maximization problem in its standard form.

 Minimize, $z = 3x_4 - 2x_2 + 4x_3$

 Subject to $x_1 - x_2 + 3x_3 \ge 1$ 2

 $2x_1 + 3x_2 - 5x_3 \ge -3$

 $4x_1 + 2x_3 \ge 2$

 $x_1, x_2, x_3 \ge 0$

P.T.O.

b) Express $(4,5)$ as a linear combination of $(1,3)$ and $(2,2)$. 3

c) Show that the feasible solution $(1,0,1,6)$ of the system of equations

$$x_1 + x_2 + x_3 = 2$$
$$x_1 - x_2 + x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 - x_4 = 0$$

is not basic. 5

4. a) Show that $x = \{x : |x| \le 2\}$ is a convex set. 2

b) Show that the intersection of two convex sets is also a convex set. 3

c) Define hyperplane. Show that the hyperplane is a convex set. 2+3

5. a) Solve the following L.P.P. by Simplex method :

Maximize, $z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \le 12$ 5

$6x_1 + 5x_2 \le 40$

$x_1, x_2 \ge 0.$

b) Solving by Big-M method, prove that the following L.P.P. has no feasible solution.

Maximize, $z = 20x_1 - x_2 + 5x_3$

Subject to $x_1 + 2x_2 + 2x_3 \le 2$ 5

$5x_1 + 6x_2 + 8x_3 = 24$

$x_1, x_2, x_3 \ge 0.$

6. a) Find the basic feasible solution of the following transportation problem by Row-Minima method : 5

	D_1	D_2	D_3	D_4	a_i
O_1	4	2	5	3	6
O_2	5	4	3	2	13
O_3	1	4	6	5	10
b_j	7	8	6	8	

b) Find the basic feasible solution of the following transportation problem by Matrix-Minima method : 5

	D_1	D_2	D_3	D_4	a_i
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	6	7	14	

UNIT-II (Marks=28)

(Numerical Methods)

Answer ***any four*** questions.

1. a) Obtain the general formula for computing the relative error of a function of *n* variables. 2

b) If $f(x) = e^{ax}$, then show that $f(0), \Delta f(0), \Delta^2 f(0)$ are in geometric progression, where Δ is forward difference operator. 3

c) Find the number of significant figures in

i) $V_A = 11.2461$ given its absolute error as 0.25×10^{-2} 1

ii) $V_T = 1.5923$ given its relative error as 0.1×10^{-3} 1

2. a) Establish Lagrange's interpolation formula. 3

b) From the following table, find the value of y when $x = 1.54$. 4

x	1.0	1.1	1.2	1.3	1.4	1.5
y	0.24197	0.21785	0.19419	0.17137	0.14973	0.12952

3. a) Extablish Newton's forward interporation formula. 3

b) Using suitable interpolation formula find a polynomial which passes through the points $(0,-12), (1,0), (3,6), (4,12)$. 4

4. a) Establish composite Simpson's one-third rule of numerical integration. 3

b) Compute, by Simpson's one-third rule,

$$\int_0^1 \frac{dx}{1+x^2}$$

taking 12 subintervals and correct upto 6-significant figures. 4

5. a) Describe the Newton-Raphson method for the numerical solution of a algebraic or a transcendental equation. 3

b) Compute the smallest positive root of the equation

$$(1+x^2)\tan x = 3x$$

by the method of fixed point iteration, correct to 5-decimal places. 4

6. a) Describe the Gauss-Jacobi's method for solving a system of three linear equations in three unknowns. 3

b) Solve by the Gauss-Jacobi's method, the system of equations

$$2x + 3y + 4z = 20$$
$$x - 4y + 7z = 14$$
$$3x + y - z = 2$$

correct upto two decimal places. 4