For Non-CBCS Students only

B.A./B.Sc.(Honours) Examination, 2019
Semester-II
Mathematics (Honours)
Core Course: BMC-23
(Differential Equation-I)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. a) Find the differential equation of the family of circles $(x-\alpha)^2+(y-\beta)^2=\gamma^2$, where α, β are arbitrary constants and γ is a fixed constant. 4

   b) Consider the first order differential equation $M(x,y)dx+N(x,y)dy=0$. If $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$ is a function of y alone, say $\varphi(y)$, then show that $e^{\int\varphi(y)dy}$ is an integrating factors of the differential equation $Mdx+Ndy=0$. Hence solve the equation $xydx+\left(2x^2+3y^2-20\right)dy=0$. 3+3

2. a) Solve the equation
$$\frac{dy}{dx}+2y\tan x=\sin x,\ \text{when } y\left(\pi/3\right)=0.$$
   Also find the maximum value of y. 5

   b) Find the orthogonal trajectories of the family of curves
   $x^{2/3}+y^{2/3}=a^{2/3}$, where a is a variable parameter. 3

   c) Solve: $\left(x^2-y^2\right)dy-xydx=0$. 2

3. a) Solve the difference equation $U_{x+2}-7U_{x+1}-8U_x=x^{(2)}2^x$. 5

   b) Solve: $x+\frac{p}{\sqrt{1+p^2}}=a,\ p\equiv\frac{dy}{dx}$. 5

4. a) Solve and find the singular solution (if any) of the differential equation
   $xp^2-2py+4x=0$. 6

   b) Reduce the differential equation
$$\left(x^2+y^2\right)(1+p)^2-2(x+y)(1+p)(x+yp)+(x+yp)^2=0$$
   to Clairaut's form by the substitution $x^2+y^2=v$ and $x+y=u$. Hence solve the equation. 4

5. a) Solve: $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x$. 5

   b) Solve the equation
$$\frac{d^2y}{dx^2}+9y=\text{cosec}\,3x,$$
   by the method of variation of parameters. 5

P.T.O.

6. a) Using the method of undetermined coefficients, solve the equation

$$\frac{d^2y}{dx^2}+4y=x^2\sin 2x.$$ 5

b) Show that if f is any solution of the differential equation

$$\frac{dy}{dx}=A(x)y^2+B(x)y+C(x),$$

then the transformation $y=f+\frac{1}{v}$ reduces the above equation to a linear equation in v. Hence solve the equation

$$\frac{dy}{dx}=(1-x)y^2+(2x-1)y-x;\text{ given solution } f(x)=1.$$ 5