Use separate answer script for each Unit

B.A./B.Sc.(Honours) Examination, 2019
Semester-II
Mathematics (Honours)
Course: BMC-22
(Vector Analysis & Number Theory)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Symbols have their usual meanings.

Unit-I (Marks-24)
Vector Analysis
Answer ***any three*** questions.

1. a) If the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar, then show that any vector $\vec{r}$ can be expressed as $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$, for some suitable scalars *x, y* and *z*. 3

   b) Prove that the area of a parallelogram with two adjacent sides $\vec{a}$ and $\vec{b}$ is $\left|\vec{a}\times\vec{b}\right|$. 2

   c) Establish the relation: $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right) = \left(\vec{a}\cdot\vec{c}\right)\left(\vec{b}\cdot\vec{d}\right) - \left(\vec{a}\cdot\vec{d}\right)\left(\vec{b}\cdot\vec{c}\right)$. 3

2. a) Define tangent vector, principal normal vector and binomial vector for a curve $\vec{r} = \vec{r}(t)\left(\equiv x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}\right)$. Find the equation for the (i) tangent, (ii) principal normal and (iii) binomial to the curve $x(t) = 3\cos t,\ y(t) = 3\sin t,\ z(t) = 4t$ at the point $t = \pi$. 1.5+4.5

   b) If $\vec{a}(t)$ be a vector of constant magnitude, then prove at $\vec{a}(t)$ and $\frac{d\vec{a}}{dt}$ are perpendicular to each other, provided $\frac{d\vec{a}}{dt} \neq 0$. 2

3. Find the vector $\vec{w}(t)$, if it satisfies the relation $\vec{w}\times\vec{r} = \cos t\,\hat{i} + \sin t\,\hat{j} + e^t\hat{k}$. Determine the constants *a*, *b*, *c* so that the vector field
   $\vec{A}(x, y, z) = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational. 4+4

4. Evaluate the line integral $\int_C \vec{F}\cdot\overrightarrow{dr}$ where $\vec{F}(\vec{r}) = (2x + y)\hat{i} + (37 - x)\hat{j}$ and the curve *C* in the *xy*-plane consists of line segments joining (0, 0) to (2, 0) and then to (2, 2). Also find the surface integral $\iint_S \vec{r}\cdot\hat{n}\,ds$ over the entire surface *S* of the region bounded by the cylinder $x^2 + z^2 = 9,\ y = 0,\ y = 8$. 4+4

5. State Stoke's theorem. Show that Green's theorem is a special case of Stoke's theorem. If $\vec{H} = \vec{\nabla}\times\vec{A}$ for any smooth vector field $\vec{A}(x, y, z)$, find the value of $\iint_S \vec{H}\cdot d\vec{S}$ for any smooth closed surface *S*. 2+6

P.T.O.

Unit-II (Marks-16)
Number Theory

Answer Question No. 6 and *any* ***one*** from the rest.

6. Answer all the questions: 6×1=6
   a) Show that an integer n is divided by 9 if the sum of digits divided by 9.
   b) Use Euler's theorem to find units digit in 3^{100}.
   c) Is $n(n+1)(2n+1)/6$ an integer for $n \geq 1$? Justify.
   d) For two relatively prime integers a, b, show that $\gcd(a+b, a-b)=1$ or 2.
   e) Find all primes of the form $n^3 - 1$.
   f) Use Mathematical induction to show that $5 \mid (3^{3n+1} + 2^{n+1})$ for all $n \in \mathbb{N}$.

7. a) Given two integers a and b, show that there exists x and y for which $c = ax + by$ iff $\gcd(a, b) \mid c$. 4
   b) If a and b be relatively prime positive integers, prove that $ax - by = c$ has infinitely many solution in positive integers. 3
   c) If p be a prime and $p \mid ab$, then show that $p \mid a$ or $p \mid b$. 3

8. a) Find a positive integer n which leaves remainder 1, 2 and 3, when divided by 2, 3 and 4 respectively. 4
   b) For a prime $p > 2$, show that $1^p + 2^p + \ldots + (p-1)^p \equiv 0 \pmod{p}$. 3
   c) Show that $\varphi(n^2) = n\varphi(n)$, $\forall$ for all $n \in \mathbb{N}$ and hence find $\varphi(256)$. 2+1
