For Non-CBCS Students only

B.A./B.Sc.(Honours) Examination, 2019
Semester-II
Mathematics (Honours)
Core Course: BMC-21
(Analysis-II)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Answer ***Question No. 1*** and ***any three*** from the rest.

1. Answer ***any five*** questions: 5×2=10
   a) Find the volume and surface area of a sphere of radius a.
   b) Find the area between the curves $y = \sin x$ and $y = \cos x$ in $[0, 2\pi]$.
   c) Find the length of the curve $x^2 + y^2 = 1$.
   d) Test for convergence of the series $\sum \frac{n^2+1}{(2n+1)(n-1)}$.
   e) Show that the series $\sum (-1)^n \frac{n}{(2n+1)^2}$ is conditionally convergent.
   f) Write down Comparison test in limit form for an infinite series.
   g) Find the envelope of the family of lines $y = mx + \sqrt{m^2+1}$.

2. a) State and prove Taylor's theorem with Cauchy's form of remainder. 1+5
   b) Expand in Macleurin's infinite series for the function $y = \sin x$ with proper justifications. 4

3. a) Find the total area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. 4
   b) Determine the total length of the cycloid
   $x = a(\theta + \sin\theta)$
   $y = a(1 - \cos\theta)$.
   Also show that $\rho^2 + s^2 = 16a^2$ for this curve. 4+2

4. a) If ρ_1 and ρ_2 be radii of curvature at the ends P and D of conjugate diameters CP and CD respectively on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = I$, where c being the centre of the ellipse, then show that $\rho_1^{2/3} + \rho_2^{2/3} = \frac{a^2+b^2}{(ab)^{2/3}}$. 5
   b) Find the volume and surface area of a cone of radius a and of height h. 5

5. a) State and prove Heine-Borel theorem. 1+5
   b) Define cover and subcover of a set $A \subset \mathbb{R}$. Give examples of a countable, finite, uncountable open cover of $\mathbb{R}$. 4

6. a) State and prove De' Alembert ratio test for the series of positive terms. 5
   b) Test for convergence of the infinite series
   $\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \ldots\ldots\ldots\ldots$ 5

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