Use separate answer scripts for each unit

B.A./B.Sc.(Honours) Examination, 2019

Semester - II

Mathematics (Allied)

Paper: BMA-21

Time: Four Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Marks: 40)

(Calculus-I)

Answer ***any four*** questions

1. a) State and prove Cauchy's mean value theorem and hence deduce Lagrange's mean value theorem. 1+3+1

   b) Expand $\sin x$ as a Maclaurin's infinite series with Lagrange form of remainder. 5

2. a) If $y=\sin^{-1}x$ then show that

   i) $\left(1-x^2\right)y_2-xy_1=0$

   ii) $\left(1-x^2\right)y_{n+2}-\left(2n+1\right)xy_{n+1}-n^2y_n=0$ 2+3

   b) Evaluate $\lim\limits_{x\to\infty}\left(\dfrac{3x}{3x+1}\right)^{3x+1}$. 3

   c) Find y_n, where $y=\log\left(x+a\right)$. 2

3. a) If $u=F\left(y-z,\, z-x,\, x-y\right)$, prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. 3

   b) Let $f\left(x,y\right)=\left(x^2+y^2\right)^{1/3}$. By using Euler's formula, show that

   $$x^2\frac{\partial^2 f}{\partial x^2}+2xy\frac{\partial^2 f}{\partial x\partial y}+y^2\frac{\partial^2 f}{\partial y^2}+\frac{2}{9}f=0.$$ 4

   c) Find the value of $\dfrac{\partial^2 f}{\partial y\partial x}$ and $\dfrac{\partial^2 f}{\partial x\partial y}$ at $\left(0,0\right)$,

   where $f\left(x,y\right)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & \text{if } \left(x,y\right)\neq\left(0,0\right) \\ 0, & \text{if } \left(x,y\right)=\left(0,0\right)\end{cases}$ 3

4. a) Obtain the reduction formula for $\displaystyle\int_0^\infty\frac{dx}{\left(1+x^2\right)^n}$ and hence evaluate $\displaystyle\int_0^\infty\frac{dx}{\left(1+x^2\right)^5}$. 6

   b) Prove that $\lim\limits_{h\to 0}\dfrac{f\left(a+h\right)-2f\left(a\right)+f\left(a-h\right)}{h^2}=f''\left(a\right)$, provided $f''\left(x\right)$ is continuous. 4

5. a) Find the radius of curvature at any point $\left(x,y\right)$ of the curve $x^{2/3}+y^{2/3}=a^{2/3}$. 5

   b) Find the asymptotes of the curve

   $$x^3-2y^3+xy\left(2x-y\right)+y\left(x-y\right)+1=0.$$ 5

6. a) Examine for extreme values of the function $x^2+y^2+\left(x+y+1\right)^2$. 5

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b) Find the surface area of the ellipsoid formed by the revolution of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\left(a>b\right)$ about its major axis. 5

Unit-II (Marks: 20)

(Three Dimensional Geometry)

Answer ***any two*** questions

1. a) Prove that the two lines whose direction cosines given by the relations $2l+2m-n=0$ and $lm+mn+nl=0$ are perpendicular to each other. 5

   b) A variable plane which is at a constant distance $3p$ from the origin O cuts the co-ordinate axes in A, B and C. Show that the locus of the centroid of the triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$. 5

2. a) Find the equation of the plane containing the line $\frac{y}{b}+\frac{z}{c}=1,\ x=0$ and parallel to the straight line $\frac{x}{a}-\frac{z}{c}=1,\ y=0$. 5

   b) Find the shortest distance between the lines $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5};\ \frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$. Also find the equation of the line of shortest distance. 5

3. a) Find the condition that the lines $x=ay+b,\ z=cy+d$ and $x=a'y+b',\ z=c'y+d'$ are at right angles. 5

   b) Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle. 5
