B.A. (Honours) Examination, 2019

Semester–II (CBCS)

Economics

Course – CC-4

(Mathematical Methods for Economics-II)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.

Answer ***any four*** questions taking two from each group

Group-A

1. (a) Find A if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$

(b) if A = $\begin{bmatrix} 1 & -2 & 3 \\ 4 & 0 & -5 \\ -3 & 2 & 4 \end{bmatrix}$ and $2A^T + 3B = 4I_3$, find B

(c) Let A be an arbitrary 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find the matrix B such that AB=I.

(d) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$. Find a 2×3 matrix with distinct non-zero entries such that AB=0.

(e) Find x and B if $B = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric 2+3+4+3+3

2. (a) Write $A = \begin{bmatrix} 4 & 5 \\ 1 & 3 \end{bmatrix}$ as the sum of a symmetric matrix B and a skew symmetric matrix C.

(b) Compute the adjoint and the inverse of the matrix $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & -1 \end{bmatrix}$

(c) Solve the equations by the matrix method

$$3x - 2y = 4$$

$$4x - 3y = 5$$

(d) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$

(e) Let $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$. Find the characteristic polynomial of A and hence A^{-1}

2+4+3+3+3

3. (a) Prove that a–1 is a factor of the determinant $\begin{vmatrix} a+1 & 2 & 3 \\ 3 & a+2 & 4 \\ 4 & 4 & a+3 \end{vmatrix}$

P.T.O.

(b) (i) Prove without expanding that $\begin{vmatrix} 3 & 4 & 5 \\ 4 & 3 & 7 \\ 5 & 7 & 5 \end{vmatrix}$

Is divisible by 23. It is given that 345, 437, 537 are each divisible by 23.

(ii) Show that $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(b+c)(c+a)(a+b)$

(c) Solve $\begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0$ 3+8+4

4. (a) Show that the adjoint of a determinant Δ of the third order is equal to Δ^2

(b) Find the value of $\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix}$

(c) Solve by Cramer's Rule

$$2x - z = 1$$

$$2x + 4y - z = 1$$

$$x - 8y - 3z = -2$$

4+6+5

Group-B

5. Consider the utility function$u(x_1, x_2) = (x_1)^{1/2}.(x_2)^{1/2}$

(a) Find the marginal utilities $\frac{\partial u}{\partial x_1}$ and $\frac{\partial u}{\partial x_2}$. Are these positive or negative?

(b) Obtain the equation of the Indifference Curve and show that it is negatively sloped.

(c) Obtain the expression for the Marginal Rate of Substitution and show that it is diminishing. 4+5+6

6. Consider the production function $F(L, K) = \left[(L)^{1/2} + (K)^{1/2}\right]^2$.

(a) Is this function homogeneous? If so of what degree?

(b) Verify the Euler's Theorem in the context of this function.

(c) Expand this function using Taylor's series.

(d) Is this function linear, strictly concave, weakly concave, strictly convex, weakly convex? 3+4+3+5

7. Suppose demand depends on own price, $p, \frac{dQ^d}{dp} < 0$, the price of a substitute good, x, $\frac{dQ^d}{dp_x} < 0$ and income, $Y, \frac{dQ^d}{dY} < 0$,

$$Q^D = f(p, p_x, Y)$$

Supply depends on own price, $\frac{dQ^s}{dp} > 0$ and input price, $w, \frac{dQ^s}{dw} < 0$

$$Q^s = f(p, w)$$

Market equilibrium is characterized by the equation

$$Q^D = Q^D = Q$$

Using the comparative statics approach find how equilibrium prices and quantities are affected if

(a) Price of the substitute good rises $\left(\frac{dQ}{dp_x}, \frac{dp}{dp_x}\right)$

(b) Income rises $\left(\frac{dQ}{dY}, \frac{dp}{dY}\right)$

(c) Input price rises $\left(\frac{dQ}{dw}, \frac{dp}{dw}\right)$ 5+5+5

8. Consider a cost minimization problem where the production function has Cobb-Douglas form Minimize Cost $C = wL + rK$ subject to the production function $Q = f(L, K) = L^{1/2}K^{1/4}$

(a) Write the first order conditions of this cost minimization problem

(b) From this find out the optimal values of the input demand functions L^*, K^*

(c) Find the minimum cost of production given input prices and output, $C = wL^* + rK^*$

(d) Verify whether the second order conditions are satisfied 2+6+2+5