

M.A. Examination, 2015

Semester – IV

Economics

Paper : XV (Group-A)

(Special Paper: Statistics & Econometrics-II)

Time: 3 Hours

Full Marks: 40

Questions are of value as indicated in the margin.

Answer **any four** questions

1. (a) What is meant by maximum likelihood estimator? 3
- (b) Obtain MLE of the parameters (μ, σ^2) for the Normal Distribution $N \sim (\mu, \sigma^2)$ with usual properties. 7
2. Derive moment generating function of Poisson Distribution and use that to find out first and second moment. 10
3. (a) What is meant by Best Linear Unbiased Estimator. 2
- (b) Let X_1, X_2 , and X_3 be three iid observations from a population with mean μ and variance σ^2 .

Consider the estimators of μ say T_1, T_2, T_3

$$\begin{aligned} T_1 &= \frac{1}{3}(x_1 + 2x_2) \\ \text{defined as } T_2 &= \frac{1}{3}(ax_1 + bx_2 + cx_3) \\ T_3 &= \frac{1}{a}(2x_1 + 2x_2 + 5x_3) \end{aligned}$$

where a, b and c are real numbers. Show that if T_2 is unbiased for μ and best among T_1, T_2 and T_3 then $(a+b+c)(a^2+b^2+c^2) \leq 15$. 8

4. Suppose $X_1, X_2, X_3, \dots$ and $Y_1, Y_2, Y_3, \dots$ are two sequences of random variables such that $X_n \xrightarrow{P} \alpha$ and $Y_n \xrightarrow{P} \beta$.

Then show that

$$X_n Y_n \xrightarrow{P} \alpha\beta \quad 10$$

5. Consider a sequence of independent Bernoulian trial with probability of success to be P ; $0 < P < 1$.

Let X_n be the number of successes in the first n trials. Then show that $\frac{X_n}{n}$ converges weakly to P . 10

P.T.O.

6. (a) State NP Lemma. 3
- (b) Consider $N(0, \sigma^2)$ distribution. Consider the testing problem $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 = \sigma_1^2 (> \sigma_0^2)$. Find the most powerful test of size $\alpha (0 < \alpha < 1)$ for it. 7
7. Let $X = (X_1 X_2 \dots X_n)^T$ be a vector of r.v's and $\mu \in R^n$ and $\Sigma_{n \times n}$ be a positive definite matrix. Then show that $\{X \sim N_n(\mu_{n \times 1}, \Sigma_{n \times n})\}$ iff $a'X \sim N(a'\mu_{n \times 1}, a'\Sigma_{n \times n}a)$, $\forall a \in R^n \ni a \neq 0_{n \times 1}$. 10
8. Write short notes: 5+5
- (a) MLE of the parameter in $R(\theta, 0); \theta < 0$; distribution.
- (b) Chebyshev's inequality.
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