Pre-Degree Examination 2017
Statistics (Elective)

Time : 3 Hours **Full Marks : 60**

Questions are of value as indicated in the margin

Question No.1 and 2 are compulsory and answer any three from the rest.

(Notations and symbols used are as usual)

1. Answer **any twelve** questions : 1×12=12

(i) If P(A) = 0, then A is always an impossible event. [write True or False.]

(ii) If for any two events A and B, $P(A \text{ or } B)=\frac{7}{10}$, $P(A \text{ and } B)=\frac{2}{5}$, $P(A \mid B)=\frac{2}{3}$, then P(A) = _____. [Fill in the blank.]

(iii) If $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{4}$, then the highest possible value of $P(A \cap B)$ is ______. [Fill in the blank]

(iv) A binomial distribution with parameters n and p is positively skew if

(a) $p>\frac{1}{2}$ (b) $p<\frac{1}{2}$ (c) $p=\frac{1}{2}$ (d) for any value of p. [Select the correct option.]

(v) The 3rd central moment of a normal (μ,σ^2) distribution is

(i) $3\sigma^2$ (ii) 0 (iii) σ^3 (iv) σ . [Select the correct option.]

(vi) If the mean of a Poisson distribution is 1.5, its mode is _____. [Fill in the blank.)

(vii) Expectation of a negative random variable is negative. [Write True or False.]

(viii) If E(XY) = E(X).E(Y), then X and Y are not necessarily independent random variables. [Write True or False.]

(ix) If E(X) = 5 and E{X(X–1)}= 44, then Var (3–2X)= _______. [Fill in the blank.]

(x) The p.d.f. of a standard normal variable t is $\varphi(t)=\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}$, $-\infty<t<\infty$.

[Write True or False]

(xi) The total number of possible samples of size 3 each drawn by SRSWOR from a population of size 8 is _______. (Fill in the blank)

(xii) If t_1 and t_2 be two estimators such that $E(t_1)=\theta_1+\theta_2$ and $E(t_2)=\theta_1-\theta_2$, find unbiased estimator of θ_1.

(xiii) Define statistic.

(xiv) In case of testing a simple hypothesis, the level of significance and probability of type -I error are same. [Write True or False.]

(xv) For a finite population of size N, S.E. of sample mean in SRSWOR is

(a) $\frac{\sigma}{\sqrt{n}}$ (b) $\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$ (c) $\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-1}{N-n}}$ (d) $\frac{\sigma}{\sqrt{N}}\sqrt{\frac{N-n}{N-1}}$

[Select the correct option]

2.Answer **any six** of the following : 2×6=12

(i) For any two events A and B, show that

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

(ii) For any two jointly distributed random variables X and Y, prove that

$E(X+Y)=E(X)+E(Y)$

(iii) Find the mean of a continuous random variable with p.d.f. $f(x)=\dfrac{1}{b-a}, a\le x\le b$

(iv) Define standard error of a statistic

(v) If $x_1, x_2, ..., x_n$ are simple random independent sample observations from $N(\theta,1)$ population, then show that $\overline{x}^2-\dfrac{1}{n}$ is an unbiased estimator if θ^2.

(vi) A coin and a die are thrown. Write down the sample space.

(vii) State the 95% confident interval of population mean of a normal (μ,σ^2) distribution.

(viii) Consider the following probability distribution :

x:	1	2	3	4
$P(X=x)$:	0.25	0.50	0.12	0.13

Find $E(X^2)$.

(ix) For a Poisson variable(X), P(X = 0) = P(X = 1), Find V(X).

3. (a) Show that Poisson distribution is a limiting case of binomial distribution under some conditions. (State the conditions clearly). 6

(b) A bag contains 5 white and 7 black balls. Find the expectation of a man who is allowed to draw two balls randomly from the bag and who is to receive one rupee for each black ball and two rupees for each white ball drawn. 6

4. (a) (i) A and B be two independent events show that A^c and B^c are independent.

(ii) There are two events A and B, such that P(A) > 0 and P(B) > 0. Prove that A and B cannot be mutually exclusive if they are independent. 3+3=6

(b) Two persons toss a perfect coin n times each. Show that the probability of their scoring the same number of heads is $\binom{2n}{n}2^{-2n}$. 6

5. (a) Derive the standard error of sample mean in case of SRSWR from a finite population. 6

(b) Show that the sample proportion is an unbiased estimator of populations proportion in case SRSWR. State its standard error. 6

6. (a) A coin is tested for unbiasedness. The hypothesis that it is unbiased is rejected if 9 or more heads are obtained out of 10. Can we take 1% as level of significance? Explain your answer. 6

(b) (i) Define level of significance, in connection with testing of hypothesis. 2×3=6

(ii) Distinguish between point estimation and interval estimation.

(iii) Distinguish between parameter and statistic, in connection with sampling.

7. Write short notes on **any two** : 6×2=12

(a) Joint distribution of two random variables.

(b) Sampling distribution of sample mean and its S.E.

(c) Frequency χ^2 and its uses in goodness of fit.

(d) Classical definition of probability and its limitations.