B.A.(Honours) Examination, 2017
Semester-IV
Integrated Mathematics & Statistics
Paper-S.1.4.5.P.6 (Subsidiary)
(Calculas-II)

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin

Answer ***any four*** from the following

1. a. Prove that the solution to a linear, autonomous, first-order differential equation, $\dot{y} + ay = b$, converges to the steady-state equilibrium, $\bar{y} = \frac{b}{a}$, no matter what the initial value, y_0, if and only if the coefficient in the differential equation is positive: $a > 0$.

   b. Let y stand for energy demand, and suppose that it grows according to $\dot{y} = 5y - 10$. If energy demand has a value of 100 at time $t = 0$, determine whether it ever converges to a steady state. 5+5

2. a. Solve the following equations and ensure that the initial conditions are satisfied

   (i) $\dot{y} = 6y - 6$ and $y(0) = 3$.

   (ii) $\dot{y} = 7$ and $y(0) = 2$.

   b. Let $p(t)$ represent the consumer price index. If the rate of inflation of the price index is constant at 5% (i.e. the growth rate of $p(t)$ is 5%), and the price index has a base value of 100 at time $t = 0$, solve for the expression showing price index as a function of time. 3+3+4

3. Find the steady state equilibrium point for the Neo-classical Model of Economic Growth (Solow Model). 10

4. a. Classify the following equation according to: (i) order, (ii) linear or nonlinear, (iii) autonomous or non-autonomous, and (iv) difference or differential equation $\ddot{y} = 2\dot{y} - \frac{y}{2} + e^t$.

   b. Increase in carbon dioxide in the earth's atmosphere have been cited as a probable cause of "global warming". Let y represent the stock of cabon dioxide and let $x > 0$ (a constant) represent the flow of carbon dioxide emissions that come from industrial activity. Assume that the dynamics of y are given by $\dot{y} = x - y^a$

   Where the term y^a represents the earth's capacity to remove carbon dioxide from atmosphere and allow its absorption elsewhere (i.e., in trees, oceans). Conduct a qualitative analysis of this model, first for the case $a > 0$ and then for the case $a < 0$. Comment on your results. 4+6

P.T.O.

5. Solve the following equations using initial conditions $y_0 = 10$ and $\dot{y}_0 = 8$.

a. $\ddot{y} + 5\dot{y} - 6y = 36$

b. $3\ddot{y} + 6\dot{y} - 9y = -18$ 5+5

6. a. If $L\{F(t)\} = f(s)$, then show that $L\{e^{at}F(t)\} = f(s-a)$, where L stands for Laplace transform.

b. Show that $L\{1\} = \frac{1}{s}, s > 0$, where L stands for Laplace transform.

c. If $L\{F(t)\} = f(s)$ and $G(t) = \begin{cases} F(t-a), t > a \\ 0 \qquad\quad , t < a, \end{cases}$

then show that $L\{G(t)\} = e^{-sa}f(s)$, where f(s) is the Laplace transform of $F(t)$.

4+1+5

7. a. If f(s) is the Fourier transform of F(x), then show that $\frac{1}{s}f\left(\frac{s}{a}\right)$ is the Fourier transform of $F(ax)$.

b. If f(s) is the Fourier transform of F(x), then show that $e^{-ias}f(s)$ is the Fourier transform of $F(x-a)$. 5+5

8. Write short note on the following:

a. Separable Equations.

b. Bernoulli's Equation. 5+5
