B.A.(Honours) Examination, 2017

Semester-II

Integrated Mathematics and Statistics

Paper: S-1.2.5.P.3 (Subsidiary)

Mathematics (Algebra-I)

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin

1. Answer ***any four*** questions 4×3=12

(a) Find the conjugate of the complex number $\frac{(2-i)}{(1-2i)^2}$

(b) Find the complex number 'z' such that $e^z = i$

(c) What do you mean by a periodic function? Show that $\cos x$ is a periodic function of period $2n\pi$ $(n = 1,2,3, \ldots \ldots)$.

(d) Find the equation whose roots are the reciprocals of the roots of the equation

$$x^3 + px^2 + qx + r = 0$$

(e) State Descartes' rule of sign. Show that the equation $x^5 + x^3 - 2x^2 + x - 2 = 0$ has at least two imaginary roots

(f) If $A = \{1,2\}$, $B = \{2,3\}$ and $C = \{3,4\}$ are three given sets then find $(A \times B) \cup (A \times C)$ and $(A \times B) \cap (A \times C)$

(g) Let $A = \{a, b, c\}$ and $B = \{x, y, z\}$. Suppose a rule $f: A \to B$ is defined as $f(a) = x, f(b) = y, f(a) = z,\ f(c) = z,$. Is '$f$' a mapping? Justify your answer.

(h) Define a binary composition on a set. Why the term binary is used?

2. Answer ***any four*** questions 4×4=16

(a) if $\sqrt[3]{x + iy} = (a + ib)$ then show that $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$

(b) Find the cube roots of unity

(c) Define the exponential function of a complex variable **z**. Express $\cos x$ and $\sin x$ in terms of the exponential values of x.

(d) Find the equation whose roots are the roots of the equation $x^4 - 3x^3 + 2x^2 + 7x - 5 = 0$, each diminished by 2.

(e) Find the condition that the equation $x^3 + px^2 + qx + r = 0$ may have two roots equal in value but opposite in signs.

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(f) Find the quotient and remainder when $x^5 - 5x^4 + 9x^3 - 6x^2 - 16x + 13$ is divided by $(x^2 - 3x + 2)$

(g) A relation $\boldsymbol{R}$ on Z (set of integers) is defined by $\boldsymbol{aRb}$ holds if $(a - b)$ is divisible by 5. Show that $\boldsymbol{R}$ is an equivalence relation.

(h) Let $f: R \to R$ be defined by $f(x) = x^2, \;\; x \in R$ (R = set of real numbers). Examine whether $\boldsymbol{f}$ is (i) injective (ii) surjective.

3. Answer ***any two*** questions:- 2×6=12

(a) Express $\boldsymbol{\cos\alpha}$ in ascending powers of $\boldsymbol{\alpha}$

(b) State De Moivre's Theorem. Suppose the theorem holds for 'n' being an integer (positive or negative). Then prove the theorem for 'n' being a fractional value.

(c) Remove the second term of the equation $x^3 + 6x^2 - 12x + 32 = 0$. Solve the transformed equation by applying Cardan's method

(d) Show that the set $G = \{1, \omega, \omega^2\}$, where $1, \omega, \omega^2$ are the cube roots of unity, forms a finite abelian group under multiplication. Also find the order of the group G.