Use separate answer scripts for each unit

# B.A./B.Sc. (Honours) Examination 2017

**Semester - VI**

**Mathematics**

**Paper: BMC-64**

**( Mechanics-IV & Mathematical Modelling )**

**Time: Three Hours** **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings

## Unit-I (Marks: 24)

**( Mechanics-IV )**

Answer ***any four*** questions

1. a) Prove that in a homogeneous fluid of density $\rho$ at rest under gravity, the difference of the pressures between any two points is proportional to the difference of their depths. 2
   b) A fine uniform cycloidal tube is held with its axis vertical and vertex downward and equal weights of two liquids occupy lengths *a* and *b*. Prove that the heights of the free surfaces of the liquids above the vertex are in the ratio $(3a+b)^2 : (3b+a)^2$. 4

2. a) Define pressure gradient of a fluid in equilibrium under the action of a given field of force *F*. If *X, Y, Z* are the components of *F* along the coordinate axes and if *F* is conservative, show that the surfaces of equi-pressure, equi-density and equi-potential energy coincide. Find also the differential equations of these surfaces. 1+1.5+1.5
   b) Prove that, if the forces per unit mass at a point (*x, y, z*) parallel to the axes are $y(a-z)$, $x(a-z)$, $xy$, the curves of equal pressure and density are rectangular hyperbolas. 2

3. a) Define the thrust of a heavy homogeneous liquid on a plane surface. 1
   b) A hollow weightless cone of vertical angle $2\alpha$ is filled with fluid, and suspended freely from a point on the rim of its base. Show that the ratio of the thrust on the base and the weight of the fluid, the cone would contain, is
   $$12\sin^2\alpha : \cos\alpha\sqrt{1+15\sin^2\alpha}.$$ 5

4. a) Define centre of pressure of a given area in a fluid. Find the depth of the centre of pressure of a plane area immersed in a fluid under the action of gravity. 1+3
   b) Show that the centre of pressure of a parallelogram immersed in a liquid with one angular point in the surface and one diagonal horizontal, lies in the other diagonal and is at a depth $\frac{7}{12}$ of the depth of the lowest point. 2

5. a) Consider the thrust on curved surfaces in a fluid in equilibrium. A solid right-circular cone of vertical angle $2\alpha$ is just immersed in water so that one generator is in the surface of the liquid. Prove that the resultant thrust on the curved surface is $\sqrt{1+3\sin^2\alpha}$ times the weight of the fluid displaced by the cone. 4
   b) State the conditions of equilibrium of a freely floating body wholly or partially immersed in a fluid. 2

6. a) Explain the term "Metacentre" for a freely floating body immersed partially in a liquid and undergone a small rotational displacement. 2

**P.T.O.**

b) Define “plane of floatation”, “surface of floatation” and “surface of buoyancy” for a freely floating body partially immersed in a liquid. 1.5+1+1.5

## Unit-II (Marks: 16)

### ( Mathematical Modelling )

Answer ***any two*** questions

1. a) What is mathematical modelling? Explain the modelling process schematically. Classify the models with illustrations and mention their basic differences. 4

   b) Show that the energy is not conserved in the spring-mass system in the presence of friction unlike that in frictionless system. 4

2. a) Convert the following ordinary differential equations to autonomous systems

   $$\ddot{y} + \left(1 + t^2\right)\dot{y} + 2ty = 0,$$

   $$\ddot{y} - \dot{y} + ty = 0,$$

   and then evaluate the Jacobian matrix for those autonomous systems. 4

   b) Analyze the behaviour of a nonlinear spring-mass system with friction and discuss its stability in all possible situations. 4

3. a) Consider an one-dimensional autonomous dynamical system $\dot{x} = f(x)$ with $f : \mathbb{R} \to \mathbb{R}$ a function which is strongly differentiable on $\mathbb{R}$. Let $x_0 \in \mathbb{R}$ be a fixed point with $f(x_0) = 0$. Then prove that $x_0$ is asymptotically stable if $f'(x_0) < 0$ and unstable if $f'(x_0) > 0$. 4

   b) Find the fixed points and determine their stability for the following systems:

   i) $\dot{x} = x - x^3$

   ii) $\dot{x} = (2x-1)^2 (3x+1)^4$. 2+2

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