Use separate answer scripts for each Unit

B.A./B.Sc. (Honours) Examination, 2017

Semester - VI

Mathematics

Paper: BMC-63

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings

Unit-I (Marks: 20)

(Tensor Calculus)

Answer ***any two*** questions

1. a) Find the spherical coordinates of a point whose cylindrical coordinates are $\left(2\sqrt{2}, \frac{\pi}{4}, 1\right)$. 2

   b) Show that the equation $x^1 = 4\cos x^2$ in spherical coordinates represents a sphere. 2

   c) A covariant vector has components xy, $2y - z^2$ and xz in rectangular cartesian coordinates. Find its covariant components in spherical polar coordinates. 6

2. a) Let a quantity $A(p, q, r, s)$ in the coordinate system $\left(x^i\right)$ be such that

   $A(p, q, r, s)B_n^{qsm} = C_{rn}^{pm}$, where B_n^{qsm} is an arbitrary tensor and C_{rn}^{pm} is a given tensor. Prove that $A(p, q, r, s)$ is a tensor. 5

   b) If A_i is a covariant vector, then show that $\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ are the components of a tensor of type (0, 2). 2

   c) If A_{ijkl} is symmetric in the first two indices from left and skew symmetric in the second and fourth indices from the left then show that $A_{ijkl} = O$. 2

   d) If A_{ij} is skew symmetric, then show that $\left(\delta_j^i \delta_p^k + \delta_p^i \delta_j^k\right) A_{ik} = 0$. 1

3. a) Calculate the Christoffel symbols

   $$\begin{Bmatrix} 1 \\ 33 \end{Bmatrix}, \begin{Bmatrix} 2 \\ 33 \end{Bmatrix}, \begin{Bmatrix} 3 \\ 32 \end{Bmatrix}, \begin{Bmatrix} 2 \\ 12 \end{Bmatrix}, \begin{Bmatrix} 3 \\ 13 \end{Bmatrix}$$

   corresponding to the metric $dS^2 = \left(dx^1\right)^2 + \left(x^1\right)^2 \left(dx^2\right)^2 + \left(x^1 \sin x^2\right)^2 \left(dx^3\right)^2$. 6

   b) If the metric in rectangular cartesian coordinates is given by $dS^2 = \left(dx^1\right)^2 + \left(dx^2\right)^2$, then find the metric in polar coordinates. 2.5

   c) If A^i and B^i are orthogonal vectors each of length l, then find the value of

   $\left(g_{hj}g_{ki} - g_{hk}g_{ji}\right) A^h B^j A^k B^i$. 1.5

Unit-II (Marks: 20)

(Mathematical Statistics)

Answer ***any two*** questions

4. a) Explain the difference between a distribution of sample and a sampling distribution. 2

   A sample of size n $\left(X : x_1, x_2, ..., x_n\right)$ is drawn from a population. How does the distribution function of the sample approximates to that of the population? What does it imply in this context? 2+1

P.T.O.

b) What is estimation of parameter?

If $f(x) = 2\gamma\sqrt{\frac{\gamma}{\pi}}x^2 e^{-\gamma x^2}$, $-\infty < x < \infty$, find the maximum likelihood estimate (MLE) for γ. Show that the estimate is *unbiased*. 1+2+2

5. a) Let T_1 and T_2 are two independent unbiased estimators of parameter θ having the same variance. Which of the estimators $\frac{1}{2}(T_1 + T_2)$ and $\frac{1}{3}(T_1 + 2T_2)$ you would prefer for estimating θ? Why? 2

b) The heights in inches of 7 students of a college, chosen randomly were as follows:
62.2, 63.1, 62.4, 63.2, 65.5, 66.2, 66.3.
Compute 95% confidence interval for the mean of the population of heights of students of the college, assuming it to be normal. 4
[Given $P(t > 2.447) = 0.025$]

c) Fit a straight line $y = a + bx$ to the following data:

x	0	1	2	3	4
y	1.0	1.8	3.3	4.5	6.3

Hence, find the difference between the actual value of y and its value obtain from the fitted line when $x = 4$. 3+1

6. a) In the context of statistical hypothesis, explain what do you mean by the following:
i) Simple and Composite hypotheses,
ii) Type I and Type II errors. 2+2

b) State Neyman-Pearson theorem on the best critical region for testing a simple hypothesis $H_o : \theta = \theta_0$ against an alternative simple hypothesis $H_o : \theta = \theta_1$. 2

c) In order to test whether a coin is perfect, the coin is tossed 5 times. The null hypothesis of the perfectness is rejected if and only if more than 4 heads are obtained. What is the probability of Type I error? Find the probability of Type II error when the corresponding probability of head is 0.2. 2+2