Use separate answer scripts for each unit

B.A./B.Sc. (Honours) Examination, 2017
Semester-VI
Mathematics
Course: BMC-61
(Analysis-VI)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and Symbols have their usual meanings.

Unit-I (Marks:20)
Complex Analysis

Answer ***any two*** questions.

1. a) Test the continuity at the origin of the function $f(z)$ defined by

$$f(z)=\frac{x^2y^2}{x^2y^2+(x-y)^2} \quad \text{for } z\neq 0$$
$$=0 \qquad \text{for } z=0.$$ 2

b) If $f(z)$ is differentiable at $\alpha\in\mathbb{C}$, then show that it is continuous at α. Show by an example with proper justification that its converse may not be true. 2

c) Verify whether $f(z)=|z|^2$ is differentiable at any point $z\in\mathbb{C}$. 2

d) If a function $f(z)=u(x,y)+iv(x,y)$ is differentiable within a region D show that $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$ at every point in D. 4

2. a) Define a harmonic function. Find a harmonic conjugate $v(x,y)$ when $u(x,y)=\sinh x\sin y$. 1+2

b) If $f(z)$ be regular in a region D and $|f(z)|$ be constant in D, then show that $f(z)$ itself is constant in D. 2

c) State and prove Cauchy's general principle of convergence of a sequence of complex numbers. 1+3

d) Show by an example that there may be a convergent series of complex numbers which is not absolutely convergent. 1

3. a) Find the Bilinear transformation which transforms the points 1, 0, ∞ in the z-plane to the points -1, i, $-i$ respectively in the w-plane. 2

b) Show that the equation $\left|\frac{z-\alpha}{z-\beta}\right|=l\,(l\neq 1)$ represents a circle with respect to which α and β are inverse points. Further show that by a Mobiüs transformation a circle transforms into a circle or a straight line. 3+2

c) State orientation principle. 1

d) Find the stereographic projection of a circle in the complex plane whose equation is given as $x^2+y^2+2gx+2fy+c=0$. 2

P.T.O.

Unit-II (Marks:20)
Metric Spaces

Answer ***any four*** questions.

1. a) Prove that $C[a,b]$, the set of all real valued continuous functions on $[a,b]$, is a metric space with respect to a metric defined by you. 3

   b) Let (X,d) be a metric space. Then show that the union of an arbitrary family of open sets in X is open in X. 2

2. a) If x and y are distinct points in a metric space (X,d), then show that there is $r>0$ such that $S_r(x)\cap S_r(y)=\varphi$. 2

   b) Show that a set F in a metric space (X,d) is closed if and only if $F'\subset F$, where F' is the derived set of F. 2

   c) Give an example to show that $(A\cap B)'$ may not be equal to $A'\cap B'$ in a metric space. 1

3. a) Show that for any subset A of a metric space (X,d), $X-\overline{A}=(X-A)^{\circ}$. [$\overline{A}$ and A° respectively denote the closure and interior of A]. 1.5

   b) Show that for any subset A of a metric space (X,d), $Diam(A)=Diam(\overline{A})$. 2.5

   c) Give an example to show that distance between two disjoint sets may be zero. 1

4. a) Let $\{x_n\},\{y_n\}$ be sequences in a metric space (X,d) such that $x_n\to x$ in X. Then show that $y_n\to x$ in X if and only if $d(x_n,y_n)\to 0$ in $\mathbb{R}$. 2

   b) Show that every Cauchy sequence in a metric space is bounded. 2

   c) Give an example of an incomplete metric space. 1

5. a) State and prove Cantor's intersection theorem. 3

   b) Show that a set A in a metric space (X,d) is nowhere dense if and only if $X-\overline{A}$ is dense in X. 1

   c) Give an example with justification of a set of first category which is nowhere dense. 1

6. a) Let (X,d) be complete metric space and $f:X\to X$ be a map. If for some positive integer n, f^n is a contraction map on X, then show that f has a unique fixed point in X. 3

   b) Give an example with justifications of a weak contraction map which is not contraction. 2

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