B.A./B.Sc. (Honours) Examination 2017
Semester-IV
Mathematics (Core)
Course: BMC-43
(Mechanics-II)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
[Symbols and Notations have their usual meanings unless otherwise stated]

Answer ***any four*** *questions.*

1. a) If $P_1, P_2, ..., P_n$ be a given system of coplanar forces acting, respectively, at the points $A_1, A_2, ..., A_n$ of a body, then show that the system of coplanar forces can be reduced to a single force R at any base point O and a single couple G. 5
   b) A system of coplanar forces has the total moments H, $2H$ and $3H$ about the points (0, 0), (0, 1) and (2, 4) respectively. Find the magnitude and the equation of the line of action of the resultant. 5

2. a) i) Define virtual displacement and virtual work for a rigid body acted upon by the force F. When a virtual displacement is said to be consistent with the geometrical conditions of the rigid body? 1.5+0.5
   ii) Find the virtual work done by the mutual action and reaction between two particles of a rigid body. 3
   b) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical respectively, prove that $\tan\phi = \frac{3}{8} + \tan\theta$. 5

3. a) If a system of bodies be in equilibrium under the action of gravitational force only, discuss the conditions of its stability and instability. Hence, show that if the system has one degree of freedom, the stable and unstable positions of equilibrium occur alternatively. 3+2
   b) A lamina in the form of a cycloid, whose generating circle is of radius a, rests on the top of another cycloid whose generating circle is of radius b, their vortices being in contact and their axes vertical. If b be the height of the c.g. of the upper cycloid above its vertex, show that the equilibrium is stable if $h < \frac{4ab}{a+b}$. 5

4. a) i) Let a rigid body be subjected to the three external forces X, Y, Z, acting along the axes at O with the resultant R and a couple G of moment $\sqrt{L^2 + M^2 + N^2}$. Prove that R is invariant but not G. 2
   ii) If any three forces P, Q and R, acting on a rigid body keep it in equilibrium, then show that they must be coplanar. 3
   b) Six forces, each equal to P, act along the edge of a cube, taken in order, which do not meet a given diagonal. Show that their resultant is a couple of moment $2\sqrt{3}\,Pa$, where a is the edge of the cube. 5

5. a) Explain what is meant by Poinsot's central axis for a system of forces in three dimensions acting on a rigid body. 3.5
   Hence define the terms Wrench, Pitch and Screw. 0.5+0.5+0.5

P.T.O.

b) Two forces, act one along the line $y = 0,\ z = 0$ and the other along the line $x = 0,\ z = c$. As the forces vary, show that the surface generated by the axis of their equivalent wrench is $\left(x^2 + y^2\right)z = cy^2$. 5

6. a) Let a heavy particle of mass m be compelled to move along a smooth cycloid, whose plane is vertical with its base AB horizontal and vertex O downwards. Discuss the motion of the particle on the cycloid. Hence determine the arc length from O to any point P in time t on the cycloid and the reaction along the normal to the curve at P. 5

b) A smooth parabolic tube is placed, vertex downwards, in a vertical plane. A particle with weight w slides down the tube from rest under the gravity. Prove that at any position, the reaction of the tube is $\dfrac{2w\left(h + a\right)}{\rho}$, where ρ is the radius of curvature, $4a$ is the latus rectum of the parabola and h is the original height of the particle above the vertex. 5

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