B.A./B.Sc. (Honours) Examination, 2017
Semester-IV
Mathematics (Core)
Course: BMC-42
(Optimization)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Symbols have their usual meanings.
Answer ***Question No. 6*** and ***three*** from the rest.

1. a) A company manufactures two products of type I and type II. Processing of these products is done on two machines A and B. Type I product requires 2 hours per unit on machine A and 6 hours per unit on machine B. Type II product requires 5 hours per unit on machine A and no time on machine B. There are 16 hours of time per day available on machine A and 20 hours on machine B. Profit gained by the manufacturer from the product of type I and type II is Rs. 10 and Rs. 50 respectively. Solve the problem graphically to find the daily production of each of the two products in order to get the maximum profit. 5

   b) Justify the statement: The dual of a dual problem is the primal problem.
   Apply the principal of duality to solve the LPP

$$\begin{aligned} \text{Maximize } & f(x_1, x_2) = 3x_1 + 4x_2 \\ \text{subject to } & x_1 + x_2 \le 10, \\ & 2x_1 + 3x_2 \le 18, \\ & x_1 \le 8, \\ & x_2 \le 6, \\ & x_1, x_2 \ge 0. \end{aligned}$$

1+4

2. a) Prove that the convex polyhedron is a convex set. Justify, whether the following set is convex and convex polyhedron:

$$X = \left\{ (x_1, x_2) \middle| x_1^2 + x_2^2 \le 1 \right\}.$$

2+(1+1)

   b) Give an example that the union of two convex sets may not be a convex set. 2

   c) What is the relation between BFS and extreme point solution of a convex set of the feasible solutions of a given LPP?
   Find the extreme point solution for the equations,

$$\begin{aligned} & 2x_1 + 6x_2 + 2x_3 = 3, \\ & 6x_1 + 4x_2 + 4x_3 = 2, \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

1+3

3. a) Prove that $x_1 = 2, x_2 = 3, x_3 = 0$ is a feasible solution but not a basic feasible solution of the set of equations

$$\begin{aligned} & 3x_1 + 5x_2 - 7x_3 = 21, \\ & 6x_1 + 10x_2 + 3x_3 = 42. \end{aligned}$$

   Reduce the feasible solution to a basic feasible solution and show that the solution is degenerate. 1+3

   b) Solve the following LPP by simplex method:

$$\begin{aligned} \text{Maximize } & f(x_1, x_2) = 2x_1 + 5x_2 \\ \text{subject to } & x_1 + 2x_2 \le 8, \\ & x_1 \le 4, \\ & 0 \le x_2 \le 3 \end{aligned}$$

   and x_1 is unrestricted in sign.
   Is the solution unique? Justify your answer. 5+1

P.T.O.

4. a) Solve the following system of linear simultaneous equations using the concept of LPP:

$$3x_1 + x_2 = 7,$$
$$x_1 + x_2 = 3.$$

4

b) How can you detect that in a transportation problem the solution is optimal? What is the criterion for the existence of multiple optimal solutions?
Solve the following balanced transportation problem by using VAM to determine the initial BFS: (1+1)+4

	D_1	D_2	D_3	D_4	a_i
O_1	9	8	5	7	12
O_2	4	6	8	7	14
O_3	5	8	9	5	16
b_j	8	18	13	3	

5. a) Under what conditions dual simplex method could be applied?
Use dual simplex method to solve the following LPP:

Maximize $f(x_1, x_2, x_3) = -2x_1 - x_3$

subject to $x_1 + x_2 - x_3 \geq 5,$

$x_1 - 2x_2 + 4x_3 \geq 8,$

$x_1, x_2, x_3 \geq 0.$

1+5

b) Find the optimal assignment to find the minimum cost for the assignment problem with the cost matrix given by: 4

		Machines			
		M_1	M_2	M_3	M_4
Jobs	J_1	10	24	30	15
	J_2	16	22	28	12
	J_3	12	20	32	10
	J_4	9	26	34	16

6. Attempt ***any two*** questions: 5×2=10

a) i) Characterize the following terms in the context of game theory:
Saddle point, Pure and mixed strategies. 1+2

ii) Show that the 2×2 game $\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$ is non-strictly determined, if

$a_1 > a_2, a_1 > a_3, a_4 > a_2$ and $a_4 > a_3$. 2

b) i) Show that the game problem $\begin{bmatrix} \alpha & -\beta \\ -\gamma & \delta \end{bmatrix}$ is fair, if $\alpha\delta - \beta\gamma = 0.$ 1

ii) Solve the following 2×4 game geometrically: 4

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	3	2	–1	4
	A_2	2	5	6	–2

c) Solve the game problem by reducing into 2×2 game problem using the dominance property

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 1 \\ 4 & 3 & 1 & 3 & 2 \\ 4 & 3 & 4 & -1 & 2 \end{bmatrix}$$

Interpret the game result with optimal strategy. 4+1

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