Use separate answer scripts for each Unit

B.A./B.Sc. (Honours) Examination, 2017
Semester-IV
Mathematics (Allied)
Course: BMA-41

Time: Four Hours **Full Marks: 60**

Questions are of value as indicated in the margin.

Unit-I (Marks-32)
(Linear Programming Problem)
Answer ***Question No. 1*** and ***Three*** from the rest.

1. Answer any two questions: 2×1=2
   a) Write a general L.P.P. in its standard form.
   b) Give an example of a convex set which has no extreme point.
   c) Let $X=\{x:|x-\alpha|=r\}$. Find the convex hull $C(X)$ of the set X.

2. a) A hospital has the following requirements for nurses:

Period	Clock time (24 hour day)	Minimum number of nurses required
1	6 A.M. - 10 A.M.	60
2	10 A.M. - 2 P.M.	70
3	2 P.M. - 6 P.M.	60
4	6 P.M. - 10 P.M.	50
5	10 P.M. - 2 A.M.	20
6	2 A.M. - 6 A.M.	30

   Nurses report to the hospital wards at the beginning of each period and work for eight consecutive hours. The hospital wants to determine the minimum number of nurses so that there may be sufficient number of nurses available for each period. Formulate this as an L.P.P. 5

   b) Solve graphically the following L.P.P.:
   Maximize $z=2x_1+x_2$
   subject to $x_1+x_2\geq 5$
   $2x_1+3x_2\leq 20$
   $4x_1+3x_2\leq 25$
   $x_1,x_2\geq 0.$ 5

3. a) Express $x=(4,5)$ as a linear combination of $a=(1,3)$ and $b=(2,2)$. 2
   b) Show that the set of vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) and (1, 1, 1) are linearly dependent. 3
   c) Show that the feasible solution (1, 0, 1, 6) of the system
   $x_1+x_2+x_3=2$
   $x_1-x_2+x_3=2$
   $2x_1+3x_2+4x_3-x_4=0$
   is not basic. 5

4. a) Show that $X=\{x:|x|\leq 2\}$ is a convex set. 3

P.T.O.

b) Determine the position of the points (1, 0, 2, –2), (0, 4, 1, 0) and (2, 2, –1, –4) with respect to the hyperplane $2x_1 + 3x_2 + x_3 - 3x_4 = 13$. 3

c) Show that a hyperplane is a convex set. 4

5. a) Solve the following LPP by simplex method:

$$\text{Maximize} \quad z = 3x_1 + 2x_2$$
$$\text{subject to} \quad 2x_1 + x_2 \le 12$$
$$6x_1 + 5x_2 \le 40$$
$$x_1, x_2 \ge 0.$$

5

b) Solve the following LPP by Big M-method:

$$\text{Maximize} \quad z = 20x_1 - x_2 + 5x_3$$
$$\text{subject to} \quad x_1 + 2x_2 + 2x_3 \le 2$$
$$5x_1 + 6x_2 + 8x_3 = 24$$
$$x_1, x_2, x_3 \ge 0.$$

5

6. a) Determine an initial basic feasible solution of the following transportation problem by Row Minima method: 5

	D_1	D_2	D_3	D_4	a_i
O_1	4	2	5	3	6
O_2	5	4	3	2	13
O_3	1	4	6	5	10
b_j	7	8	6	8	

(b) Find the basic feasible solution of the following transportation problem by Matrix Minima method: 5

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	2	11	10	3	7	4
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
b_j	3	3	4	5	6	

Unit-II (Marks-28)

(Numerical Methods)

Answer ***any four*** questions.

7. a) Obtain the general formula for computing the relative error of a function of n variables. 2

b) If $\Delta r = \Delta h = 0.01$, find the absolute and relative errors correct upto two significant figures for $V = \frac{1}{3}\pi r^2 h$ when $r = 2$ and $h = 3$. 2

c) Establish Newton's Backward interpolation formula without error term. 3

8. a) Prove that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$. 2

b) Prove that $D = \frac{1}{h}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \ldots\right]$ where $D \equiv \frac{d}{dx}$ and Δ is a forward difference operator. 2

c) Using Newton's forward and backward interpolation formula, find the formula for computing the values of the first derivatives of an interpolating polynomial of degree n at x_0 and x_n. 3

9. a) What do you mean by polynomial interpolation for a given set of values of a function. 1.5

b) Using suitable interpolation formula find the polynomial which passes through the points (–2, 25), (1, –8), (2, –15) and (4, –25). 3

c) Show that the form of Lagrangian function remains invariant under linear transformation. 2.5

10. a) Derive composite Trapezoidal rule for numerical integration $\int_a^b f(x)dx$. 3

b) Evaluate $\int_0^1 \left(4x - 3x^2\right)dx$ by Simpson's one-third rule, taking 10 equal sub intervals. Compute the exact integral and find its absolute and relative errors. 4

11. a) Describe the Newton-Raphson method for the solution of an algebraic or transcendental equation. 3

b) Determine the root of $3x - \cos x = 1$ correct up to 3-significant figures by fixed point iteration method. 4

12. a) What is meant by diagonally dominant matrix? 1

b) Describe briefly Gauss-Seidel method for solving a system of three linear algebraic equations in three unknowns. 2

c) Solve the following system of equations by Gauss-Jacobi iteration method:

$$x_1 + 15x_2 - x_3 = 29.92$$
$$2x_1 + x_2 - 20x_3 = -55.64$$
$$20x_1 - x_2 + x_3 = 23.28$$

correct up to two decimal places. 4

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