B.A./B.Sc. (Honours) Examination, 2017
Semester-II
Mathematics (Core)
Course: BMC-23
(Differential Equation-I)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***Question No. 1*** and any ***three*** from the rest.

1. Answer ***any ten*** questions from the following: $10\times1=10$
   a) Find the conditions for which the general solution of the differential equation with constant coefficients $\frac{d^2y}{dx^2}+a\frac{dy}{dx}+by=0$ approaches zero as $x\to\infty$.
   b) Find $y(4)$, where $y(x)$ is the solution of the differential equation
   $$\frac{d}{dx}\left(x\frac{dy}{dx}\right)=x;\ y(1)=0,\ y'(1)=0.$$
   c) Solve the following nonlinear differential equation:
   $$\left(\frac{dy}{dx}\right)^2+5\left(\frac{dy}{dx}\right)+6=0.$$
   d) Find the orthogonal trajectories of the family of straight lines $y=kx$, k is some parameter.
   e) Solve the following differential equation by inspection method:
   $$xdy+ydx+\frac{xdy-ydx}{x^2+y^2}=0.$$
   f) Find the differential equation, whose solution is of the form
   $$y(x)=a+bx+cx^2+de^{-11x},$$
   where *a, b, c* and *d* are arbitrary constants.
   g) Solve the following differential equation by method of separation of variables:
   $$\log\left(\frac{dy}{dx}\right)=ax+by.$$
   h) Find the value of n for which the differential equation
   $\left(e^x+\frac{nx}{y}\right)dx-\frac{x^2}{y^2}dy=0$ is exact and hence solve the resulting equation.
   i) Find the function $v(x)$, where $v(x)$ satisfies the differential equation
   $$\left(D^2+1\right)^5(D-5)^2v(x)=0;\ D\equiv\frac{d}{dx}.$$
   j) Find the function $f(x)$, where $\Delta^{-1}\left(x^3+x^2\right)=f(x)$.
   k) Find the Wronskian of linearly independent solutions of the differential equation
   $$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=0.$$

l) If an integral curve of the differential equation $(y)\frac{dy}{dx}=1$ passes through (0, 0) and $(\alpha, 1)$, then find the value of α.

m) Find the value of u_5, where u_x is the solution of the difference equation

$$u_{x+2}-u_{x+1}-6u_x=0;$$

where $u_1=1, u_2=13$.

2. a) Show that the solution of the differential equation $\frac{dy}{dx}+P(x)y=Q(x)$ is

$$y(x)=e^{-\int Pdx}\int Qe^{\int Pdx}dx+Ce^{-\int Pdx},$$

where C is an arbitrary constant. Hence solve the equation

$$\frac{dy}{dx}+P(x)y=Q(x) \text{ with } y(0)=3,$$

where $P(x)=\begin{cases} 2 & 0\le x\le 1 \\ -\frac{2}{x} & x>1 \end{cases}$ and $Q(x)=4x$. 2+4

b) Solve the second order linear differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$. 3

c) Use the method of isoclines to sketch some of the solution curves of the equation $\frac{dy}{dx}=x^2+y^2$. 1

3. a) Find a family of oblique trajectories that intersect the family of hyperbolas $xy=c$ at an angle 45°. 4

b) Solve the nonlinear differential equation $y=2px+p^3y^2$. 3

c) Solve the differential equation

$$x\frac{dy}{dx}-y=(x-1)\left(\frac{d^2y}{dx^2}-x+1\right),$$

where x and e^x are two linearly independent solutions of the corresponding homogeneous equation. 3

4. a) Transform the differential equation $x^2p^2+2pxy+py^2+y^2=0$ to Clairaut's form by the substitutions $y=u$ and $xy=v$ and hence solve the equation. 3

b) Find the singular solution of the differential equation $xp^2=(x-a)^2$. Also find the tac-locus, cusp-locus and node-locus, if there by any. 4

c) Reduce the differential equation $\frac{dy}{dx}=1-x(y-x)-x^5(y-x)^5$ to a linear form. 1

d) Radium disappears at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain after the end of 100 years. 2

5. a) Consider the first order differential equation

$$M(x, y)dx+N(x, y)dy=0.$$

If $\frac{1}{\psi(M, N)}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=\varphi(x+y)$ is a function of $(x+y)$ alone and $e^{\int\varphi(x+y)d(x+y)}$ is an integrating factor of the above differential equation, then find the function $\psi(M, N)$.

Hence solve the equation

$$\left(2xy-y^2-y\right)dx+\left(2xy-x^2-x\right)dy=0.$$ 5

b) Solve the differential equation $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^3e^{2x}+xe^{2x}$ by the method of undetermined coefficients. 3

c) Find the differential equation of all circles whose centres are on the y-axis. 2

6. a) Solve the difference equation

$$u_{x+2}-7u_{x+1}-8u_x=x^{(2)}2^x.$$ 5

b) Find an integrating factor of the differential equation

$$\left(-3x^{-1}-2y^4\right)dx+\left(-3y^{-1}+xy^3\right)dy=0.$$ 3

c) Show that in a first order an first degree homogeneous differential equation $\frac{dy}{dx}=\frac{f(x, y)}{g(x, y)}$, the variables can be separated by the substitution $y=vx$. 2

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