B.A./B.Sc. (Honours) Examination, 2017
Semester-II
Mathematics (Core)
Course: BMC-21
(Analysis-II)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***Question No. 1*** and ***any three*** from the rest.

1. Answer ***any five*** questions: 5×2=10
   a) Find volume and surface area of a sphere of radius a.
   b) Find asymptote of the curve $x^3 + y^3 = 3axy$.
   c) Suppose $A(a, a')$ and $B(b, b')$ are two points on a curve $y = f(x)$. Show that entire length of the arc $AB = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\,dx$.
   d) Find the area between the curves $y = \sin x$ and $y = \cos x$ in $[0, \pi]$.
   e) Show that $\limsup(a_n + b_n) \le \limsup a_n + \limsup b_n$.
   f) Show that the sequence $\left\{\sum_{i=1}^{n} \frac{1}{i}\right\}$ is an unbounded sequence.
   g) State Leibintz test for alternating series. Show that $\sum(-1)^n \frac{1}{\sqrt{n}}$ is conditionally convergent.

2. a) State and prove the necessary conditions for a power series representation of a function. 4
   b) Expand $\sin x$ in power series. 3
   c) State Taylor's theorem with Cauchy's form of remainder. Show that it is a generalisation of Lagrange's mean value theorem. 3

3. a) Define envelope of a family of curves. Show that the envelope touches every member of the family at characteristic points. 1+4
   b) Find the envelope of the family of circles whose centres lie on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and which passes through its centre. 5

4. a) If ρ_1 and ρ_2 be the radii of curvatures at the ends P and D of conjugate diameters CP and CD respectively on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = I$, where C being the centre of the ellipse, then show that $\rho_1^{2/3} + \rho_2^{2/3} = \frac{a^2 + b^2}{(ab)^{2/3}}$. 5
   b) Give a rough sketch of the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, and find the length of its one arc. 5

5. a) Show that a Cauchy sequence is convergent. 4

P.T.O.

b) Define upper and lower limits of a sequence. Show that they are equal if and only if the sequence is convergent. 2+4

6. a) Find the total area bounded by the cardioid $r = a(1-\cos\theta)$. 4

b) State comparison test in limit form for a series of positive terms and prove it. 3

c) What is a *p*-series? Show that a *p*-series diverges for $p \geq I$. 3

7. a) State and prove Kummer's test for the series of positive terms. Show that Raabe's test can be derived from Kummer's test. 4+2

b) Show that the series

$$\frac{1^2}{2^2} + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2} + \frac{1^2 \cdot 3^2 \cdot 5^2}{2^2 \cdot 4^2 \cdot 6^2} + \text{...........}$$ is divergent. 4

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