Use separate answer scripts for each unit

B.A./B.Sc. (Honours) Examination, 2017

Semester - II

Mathematics (Allied)

Paper: BMA-21

Time: Four Hours **Full Marks: 60**

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Unit-I (Marks: 40)

(Calculus-I)

Answer ***any four*** questions

1. a) If $\log y = \tan^{-1} x,$ then prove that

   i) $\left(1+x^2\right)y_2 + (2x-1)y_1 = 0$

   ii) $\left(1+x^2\right)y_{n+2} + (2nx+2x-1)y_{n+1} + n(n+1)y_n = 0.$ 4

   b) If $f(x) = \tan x$ and n is a +ve integer, prove that

   $$f^n(0) - {}^nC_2 f^{n-2}(0) + {}^nC_4 f^{n-4}(0) - \ldots = \sin\frac{n\pi}{2}.$$ 3

   c) Evaluate $\lim\limits_{x\to\infty}\left(\dfrac{3x}{3x+1}\right)^{3x+1}.$ 3

2. a) State and prove Cauchy's mean value theorem and from it deduce Lagrange's mean value theorem. 1+3+1

   b) If $p(x)$ is a polynomial of degree (>1) and $K \in \mathbb{R},$ prove that between two real roots of $p(x) = 0$ there is a real root of $p'(x) + Kp(x) = 0.$ 2

   c) Expand $\sin x$ as a Maclaurin's infinite series. 3

3. a) For the function $f(x,y) = \left(\dfrac{y-x}{y+x}\right)\left(\dfrac{1+x}{1+y}\right),$ show that the repeated limit exists but the double limit does not exist at (0, 0). 3

   b) If $u = F(y-z,\, z-x,\, x-y),$ prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0.$ 3

   c) If $f(x,y) = \left(x^2+y^2\right)^{\frac{1}{3}}$, use Euler's formula to show that

   $$x^2\frac{\partial f^2}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} + \frac{2}{9}f = 0.$$ 4

4. a) Show that $\dfrac{\partial^2 f}{\partial y \partial x} \neq \dfrac{\partial^2 f}{\partial x \partial y}$ at $(0, 0)$ where

   $$f(x,y) = xy\frac{x^2-y^2}{x^2+y^2}, \quad \text{if } (x,y) \neq (0,0)$$
   $$= 0, \qquad (x,y) = (0,0).$$ 5

P.T.O.

b) Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$, where n is a +ve integer and greater than 1.

Hence evaluate $\int_0^3 \sqrt{\dfrac{x^3}{(3-x)}}\,dx.$ 3+2

5. a) Find the radius of curvature at the point (r, θ) on the cardioide $r = a(1-\cos\theta)$ and show that it varies as $\sqrt{r}$. 3

b) Find the asymptotes of the cubic $x^3 - 2y^3 + xy(2x-y) + y(x-y) + 1 = 0$. 4

c) Prove that the curve $y = \cos^{-1} x$ is everywhere concave to the y-axis excepting where it crosses the y-axis. Also, determine its points of inflexion, if any. 3

6. a) Examine for extreme values of the function $x^2 + y^2 + (x+y+1)^2$. 3

b) Find the are of the region bounded by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. 4

c) Prove that the surface area of the ellipsoid formed by the revolution of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1\,(a > b)$ round its major axis is $2\pi ab\left\{\sqrt{1-e^2} + e^{-1}\sin^{-1} e\right\}$ square unit. 3

Unit-II (Marks: 20)

(Three Dimensional Geometry)

Answer ***any two*** questions

1. a) Find the direction cosines of the line which is equally inclined to the axes. 2

b) Prove that the two lines whose direction cosines are given by the relations $2l + 2m - n = 0$ and $lm + mn + nl = 0$ are perpendicular to each other. 4

c) Find the equation of the plane which bisects the line joining (1, 2, 3) and (3, 4, 5) at right angles and show that it makes equal intercepts on the co-ordinate axes. 4

2. a) Express the equation of the plane $2x - 3y + 4z - 5 = 0$ in normal form. 2

b) Find the shortest distance between the lines

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5};\ \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}.$$

Also find the equation of the line of shortest distance. 4

c) Find the equation of the perpendicular from the point (3, –1, 11) to the line $\dfrac{x}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$. Obtain the foot of the perpendicular. 4

3. a) A plane passes through a fixed point (p, q, r) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere OABC is $\dfrac{p}{x} + \dfrac{q}{y} + \dfrac{r}{z} = 2,$ where O being the centre of the sphere. 4

b) Find the co-ordinates of the centre and radius of the circle $x - 2y - 2z + 7 = 0;$ $x^2 + y^2 + z^2 - 2x + 6y + 4z - 35 = 0.$ 3

c) Obtain the equation of the circle that lies on the sphere $x^2 + y^2 + z^2 - 2x + 2y - 4z + 3 = 0$ and having its centre at the point (2, 2, –3). 3