M.Sc. Examination, 2019
Semester-II
Computer Science
Course : MCSC-21
(Design and Analysis of Algorithm)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

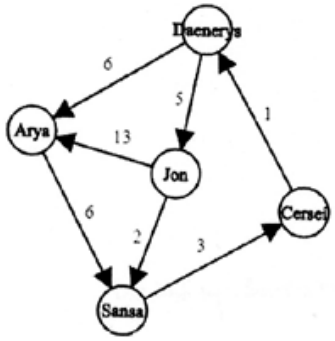
Answer Question No.**1** and **any three** from the rest.

1. Answer **any five** questions. 2×5=10
   a) Discuss Optimal Substructure property.
   b) Use the master method to give tight asymptotic bounds for the recurrence $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$.
   c) If $f(n) = \sum_{i=1}^{n} i^2$, then prove that $f(n) \in \theta(n^3)$.
   d) What are the disadvantages of Dijkstra's algorithm to find shortest path between a pair of vertices.
   e) Use the master method to give tight asymptotic bounds for the recurrence $T(n) = 2T\left(\frac{n}{2}\right) + \theta(n)$.
   f) What is a longest common sub-sequence (LCS)?
   g) Let $f(n)$ and $g(n)$ be asymptotically non-negative functions. Using the basic definition of θ notation, prove that $\max(f(n), g(n)) = \theta(f(n) + g(n))$.
2. Suppose we have a dictionary D, which is an array of n unique strings, sorted in alphabetically order. A procedure COMPARE-STRINGS (x, y) has been provided to compare two strings x and y in θ (1) time, and it returns True if x comes before y, and False if y comes before x in lexicographic order. We are also given an array A, which contains $n-1$ of the n strings in D, but A is not sorted. Now, you need to develop an algorithm to find the string that is not present in A.
   a) Design a divide-and-conquer algorithm that will find the missing string in $O(n)$ time. You can assume that we can utilize the Partition algorithm that is used by QuickSort, by modifying it to compare strings instead of comparing numbers (you do not need to show how to do the modification). Write the pseudo-code of your algorithm and explain how it works.
   b) Show that the runtime of your algorithm is $O(n)$. 8+2=10
3. a) What is convex hull?
   b) With example explain Graham's Scan algorithm to find the convex hull of a given set of two dimensional points.
   c) What is the time and space complexity of Graham's Scan algorithm? 2+6+2=10
4. Although you have thousands of friends in your Facebook, only five of them live in your home town. They are Jon, Daenerys, Arya, Sansa, and Cersi. You know a few roads that connect some of their houses, and you know the lengths of those roads. Look at the graph below, it shows paths from one friend to another with corresponding distances.

P.T.O.

a) Develop an algorithm to find the shortest distance between each pair of friends.

b) Analyse your proposed algorithm to measure the time and space complexity. 8+2=10



5. Given a sequence of matrices A_1, A_2, A_3,…A_n, you need to find the best way (using the minimal number of multiplications) to compute their product.

   a) Show that there is not only way to compute the product.

   b) Develop a Dynamic Programming based solution for the given problem. Use an example to demonstrate correctness of your solution.

   c) Analyse your proposed solution to measure the time and space complexity. 2+(4+2)+2=10

6. a) Write down the definition for Minimum Spanning Forest (MSF).

   b) Develop an algorithm for detecting MSF from a given graph.

   c) State best and worst case scenarios for this algorithm.

   d) Analyze the algorithm to calculate the time complexity for these two scenario. 2+4+2+2=10

7. State 0-1 Knapsack problem and prove that this problem is NP-complete. State any assumption you have that affects your proof. 1+9=10
