M.Sc. Examination, 2019

Semester - IV

Physics

Special Course: MPC-466

(Cosmology)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions

1. (a) State Hubble's law.

(b) Considering the homogeneous and isotropic FRW metric, find out the non-zero Christoffel symbols corresponding to time coordinate.

(c) For a matter dominated universe ($k \neq 0$), show that

$$H^2 = H_0^2\ (1+z)^2(1+2q_0 z)$$

(d) An object has luminosity distance d_L and angular diameter distance d_A. Show that the ratio $\frac{d_A}{d_L} = \frac{1}{(1+z)^2}$ is independent of cosmological model. 1+3+3+3=10

2. (a) Using the spherical collapse approximation show that the density contrast of a spherically symmetric overdense region in an Einstein-de Sitter Universe will be $\delta_{NL} = 177$ at virialization.

(b) Calculate the comoving distance to a galaxy for a spatially flat $\Omega_{m0} = 1$ universe.

(c) Show that for the universe to accelerate, it should have a component with large negative pressure. 6+2+2

3. (a) Define cosmological particle horizon. Find out the particle horizon for a FRW universe with $\Omega_{\Lambda 0} = 1$.

(b) Briefly describe the horizon problem of standard Big Bang Cosmology. How does inflation solve this problem?

(c) Show that the variance of a density field smoothed with a spherical top hat filter of size R is,

$$\sigma_R^2 = \int \frac{d^3k}{(2\pi)^3} P(k)\, 9 \left[\frac{\sin(kR) - kR\cos(kR)}{(kR)^3}\right]^2$$

where $P(k)$ is the power spectrum of the density fluctuations. (1+2)+(1+2)+4

4. (a) (i) Solve the Einstein's equation for a matter dominated FRW universe with $k = +1$ to obtain the age of the universe.

(ii) Hence determine the age of the universe for $q_0 = 1$ and $H_0 = 70\ km/s/Mpc$.

(b) (i) Why there is a bent in the matter power spectrum?

(ii) Show that the matter power spectrum is expected to bend at a wavenumber $k = \frac{\pi}{2(\sqrt{2}-1)} \frac{\Omega_{m0}}{\sqrt{\Omega_{r0}}} \frac{H_0}{c}$ where H_0 is the present value of the Hubble parameter and Ω_{m0} and Ω_{r0} are the present value of density parameters for matter and radiation respectively.

(3+2)+(2+3)

5. (a) Consider the Lagrangian for a scalar field as

$$L_\phi = \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

and the corresponding energy momentum tensor as

$$T_{\mu\nu} = -\frac{1}{2} g_{\mu\nu} g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi) g_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi.$$

(i) Calculate the components of energy momentum tensor for the scalar field.

(ii) Using the Einstein's field equations for a FRW universe filled with a scalar field component arrive at the equation of motion for the scalar field given by

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0.$$

(iii) Using the given Lagrangian L_ϕ, show that the above equation is the Klein-Gordon equation for the scalar field.

P.T.O.

(b) Explain how the expansion of the Universe preserves the thermal spectrum for CMB ?

(c) Show that an observer moving with velocity v in a direction θ relative to the CMB sees the rest frame blackbody spectrum with temperature T as a blackbody spectrum with temperature $T' = T\,(1 + \frac{v}{c}\cos\theta)$. (2+2+2)+2+2

6.(a) Given the governing equation for the growth of the density perturbations,

$$\frac{\partial^2\delta}{\partial t^2} + 2H\frac{\partial\delta}{\partial t} = 4\pi G\rho_m\delta.$$

Show that the growing mode in an Universe with only matter and cosmological constant is

$$D(a) = \frac{5}{2}\Omega_{m0}X^{\frac{1}{2}}(a)\int_0^a \frac{da'}{a'^3X^{\frac{3}{2}}(a')}$$

, where $X(a) = [\Omega_{m0}a^{-3} + \Omega_{\Lambda 0}]$.

(b) (i) Draw Hubble's tuning fork diagram and hence describe Hubble's morphological classification of galaxies.

(ii) Find out the ratio of the semi-major and semi-minor axes for the galaxy $E7$. 6+(2+2)

7. (a) Show that the curl of the peculiar velocity field $\vec{\nabla}\times\vec{v}$ decays away with the expansion of the Universe.

(b) The present day CMB temperature is $T_{CMB,0} = 2.725K$. What is the ratio of number density of CMB photons now and at redshift z=1000?

(c) Explain how rotation curve of spiral galaxies account for the existence of dark matter in our universe. 4+2+4