M.Sc. Examination, 2019

Semester - II

Physics

Paper: MPC-24

(Quantum Mechanics-II)

Time: Three Hours **Full Marks: 40**

Questions are of value as indicated in the margin.

Answer ***any four*** questions

Symbols bear their usual meanings.

1. (a) Define angular momentum in quantum mechanics. Show that $\mathbf{J} \times \mathbf{J} = i\hbar\mathbf{J}$. Compare the definition and the result with classical ones.

   (b) Define ladder operators. Prove the following relations –

   i. $[J_z, J_\pm] = \pm\hbar J_\pm$

   ii. $\left[J^2, J_\pm\right] = 0$

   iii. $[J_+, J_-] = 2\hbar J_z$

   $((1+2+1)+2\times 3)$

2. (a) If $\vec{A}$ and $\vec{B}$ are vector operators which commute with $\vec{\sigma}$, show that

$$\left(\vec{\sigma}\cdot\vec{A}\right)\left(\vec{\sigma}\cdot\vec{B}\right) = \vec{A}\cdot\vec{B} + i\vec{\sigma}\cdot\left(\vec{A}\cdot\vec{B}\right)$$

   (b) i. Show that for a rotation of a spinless particle over an infinitesimal angle $\delta\phi$

$$R_n(\delta\phi) = 1 + (i\hbar)^{-1}\vec{n}\cdot\vec{L}\delta\phi$$

   ii. Show further that for a finite rotation of an angle ϕ, the following relation hold

$$R_n(\phi) = \exp\left[-\frac{i}{\hbar}\phi\ \hat{n}\cdot\vec{L}\right]$$

   where $\hat{n}$ is the axis of rotation. (4+4+2)

3. (a) Compare between Schrödinger, Heisenberg and Dirac pictures of quantum dynamics.

   (b) Find the Dyson series of the time-evolution operators.

   (c) Find the classical analogue of Schrödinger equation. (4+4+2)

4. (a) Find the transiton probability for a Harmonic perturbation.

   (b) Find the Fermi's Golden rule for a transiton of a system to a continuum. (5+5)

5. (a) Find the differntial cross-section of a scattering from a target using first Born Approximation.

   (b) Find the scattering cross-section to the first Born approximation for a potential $V(r) = V_0 \exp[-\frac{r}{R}]$ where R is defined to be a constant. (6+4)

6. (a) Establish KFG equation in covariant form stating proper postulates. Discuss whether a single particle interpretation of KFG equation exists. Support your claim with suitable mathematics.

   (b) Show that Dirac γ-matrices obey Clifford algebra.

   (c) Show that Dirac γ-matrices are traceless. (3+3)+2+2

P.T.O.

7. (a) Show that the Dirac equation, in presence of electromagnetic field, rduces to Pauli equation in non-relativistic limit.

   (b) Explain explicitly why the solution of Dirac equation is not unique.

   (c) Explain the "negative energy" solutions of Dirac equation. (5+3+2)
