Five Year Integrated M.Sc. Examination, 2017

Semester-IV

Course: MT-2-4-2

(Mathematical Theory of Probability & Statistics)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.

Answer ***any six*** questions.

1. a) State and prove Boole's inequality in probability. 1+4
   b) Show that the probability of occurrence of only one of the events A and B is $P(A)+P(B)-2P(AB)$. 2
   c) A coin is tossed $m+n$ times $(m>n)$. Show that the probability of exactly m consecutive heads is $(n+3)/2^{m+2}$. 3

2. a) If $\{A_n\}$ is a monotonic sequence of events, then prove that $P(\lim A_n)=\lim P(A_n)$. 6
   b) If the random variable k is uniformly distributed over (0, 5), then what is the probability that the roots of the equation $4x^2+4kx+(k+2)=0$ are real? 4

3. a) If X and Y are independent random variables, then prove that $P(a<X\le b,\, c<Y\le d)=P(a<X\le b)\cdot P(c<Y\le d)$, where $a<b,\, c<d$. 3
   b) The joint probability density function of two random variable X and Y is $k(1-x-y)$ inside the triangle formed by axes and the line $x+y=1$ and zero elsewhere. Find the value of k and calculate $P\left(X<\frac{1}{2},\, Y>\frac{1}{4}\right)$. Also find the marginal distributions of X of Y and determine whether the random variables are independent or not. 7

4. a) A line of length a units is divided into two parts. If the first part is of length X, find $E(X)$, var(X) and $E\{X(a-X)\}$. 6
   b) Find the characteristic function of Gamma distribution. 4

5. a) What is skewness of a probability distribution? 1
   b) Find the coefficient of skewness γ_1 of the probability distribution whose density function is defined as follows:
   $$f(x)=\frac{3}{4}x(2-x),\ 0<x<2.$$ 5
   c) For the binomial $(n,\, p)$ distribution, prove that
   $$\mu_{k+1}=p(1-p)\left(nk\mu_{k-1}+\frac{d\mu_k}{dp}\right).$$ 4

6. a) If X is a random variable with $f(x)=0$, when $x<0$, and with $E(x)=\mu$, then prove that for any $\alpha>0$, $P(X\ge\alpha)\le \mu/\alpha$. 3
   b) State and prove Tchebycheff inequality in probability. 1+4
   c) Use Tchebycheff's inequality to prove that $P(X=\mu)=1$ if $Var(X)=0$. 2

P.T.O.

7. a) State and prove Baye's theorem in probability. 1+5
   b) There are 3 true coins and 1 false coin with 'head' on both sides. A coin is chosen at random and tossed 4 times. If 'head' occurs all the 4 times, what is the probability that the false coin has been chosen and used? 4

8. a) What is estimation? 1
   b) What are the criteria for a good estimator? Discuss. 3
   c) State the important properties of maximum likelihood estimators. 2
   d) Find the maximum likelihood estimate of the parameter μ of a Normal population $N\left(\mu, \sigma^2\right)$, when the parameter σ^2 is known. Show that this estimator is unbiased. 3+1

9. a) Show that if the number of trials is very large, the probability of success is extremely small and the mean is finite, then Poisson distribution may be obtained as a limiting case of Binomial distribution. 5
   b) Let X be a random variable with density function:

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

   Then find the probability density function of $Y = 8X^3$. 3
   c) What is the frequency definition of probability? Why this definition is better than the classical definition? What are the drawbacks of this definition? 2

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