Five Year Integrated M.Sc. Examination, 2017

Semester - IV

Course: MT-2-4-1 (Old)

(Partial Differential Equations)

Time: Three Hours **Full Marks: 60**

Questions are of value as indicated in the margin.

Answer ***any four*** questions.

1. a) Define complete integral and general integral of partial differential equations. 4

   b) Find the partial differential equation arising from $\varphi\left(x+y+z,\, x^2+y^2-z^2\right)=0$ where φ denotes an arbitrary function. 5

   c) Find the complete integral and the singular solution if it exists for the non-linear equation
   $$pq+x(2y+1)p+\left(y^2+y\right)q-(2y+1)z=0.$$ 6

2. Find the general solution of the following PDE (***any three***): 3x5

   a) $xp+2yq=(x+y)z$

   b) $x\left(z^2-y^2\right)p+y\left(x^2-z^2\right)q=z\left(y^2-x^2\right)$

   c) $z\left(x_2x_3p_1+x_1x_3p_2+x_1x_2p_3\right)=x_1x_2x_3$

   d) $p^3-2pq+3q=5$

   e) $p^2z^2+q^2=1$

3. a) Solve the following partial differential equation
   $$\left(D^2-2DD'+2D'-D+2D'\right)z=(2-4x)e^{-y}.$$ 6

   b) Deduce the equation $z_{xx}=x^2z_{yy}$ to canonical form and determine its nature. 6

   c) Find the characteristics or characteristic curve of the equation $3u_{xx}-10u_{xy}+3u_{yy}=0.$ 3

4. a) Using separation of variable method solve the equation of the transverse vibration of a string, $u_{tt}=c^2u_{xx}$ $0\le x\le l,\, t\ge 0$ subject to $u(0,t)=u(l,t)=0\ \forall t\ge 0$ and $u(x,0)=Q_0(x)$ and $u_t(x,0)=Q_1(x)$ for $0\le x\le l.$ 8

   b) Define convolution for Laplace transforms. State and prove convolution theorem for inverse Laplace transform. 2+5

5. a) Define Laplace transform. What do you mean "Function of class A". 5

   b) If $L\{F(t)\}=f(s)$ then prove that $L\left\{\int_0^t F(x)\,dx\right\}=\dfrac{f(s)}{s}.$

   Hence show that $L\left[\int_0^t \sin 2u\,du\right]=\dfrac{2}{s\left(s^2+4\right)}.$ 5

   c) Find the inverse Laplace transform of the following functions (***any two***): 5

   (i) $f(s)=\dfrac{s+1}{s^2+6s+25}$, (ii) $f(s)=\dfrac{2s^2+5s-4}{s^3+s^2-2s}$, (iii) $f(s)=\dfrac{1}{s^2+8s+16}$

6. a) Solve the following differential equation with the help of Laplace transform method: 7
   $$\left(D^2+n^2\right)y=\sin nt,\ \text{if } y=Dy=0 \text{ when } t=0.$$

P.T.O.

b) Solve by Laplace transform technique

$$\frac{\partial^2 y}{\partial t^2} = 9\frac{\partial^2 y}{\partial x^2} \text{ where } y(0, t) = 0;\ y(2, t) = 0 \text{ and } y(x, 0) = 20\sin 2\pi x,\ y_t(x, 0) = 0.$$ 8

7. a) Use convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$. 3

b) Show that $\int_0^\infty \cos(x^2)\,dx = \frac{1}{2}\left(\pi/2\right)^{1/2}$. 4

c) State and prove change of scale property on Inverse Laplace transform. 3

d) Solve the Laplace transform method:

$$\left(D^2 + 4D + 8\right)y = 0;\ \ y(0) = 2,\ y'(0) = 2.$$ 5
